Oppgave 1

$$h'(1,1) = g'(\vec{F}(1,1))\vec{F}'(1,1)$$
$$= g'(2,3)\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 3 \end{pmatrix}$$
$$\frac{\partial h}{\partial x}(1,1) = \frac{\partial h}{\partial y}(1,1) = 3$$

Svaralternativ (a)

Oppgave 2

$$\vec{r}(t) = (\cos t, \sin t, t^2)$$
$$\vec{v}(t) = (-\sin t, \cos t, 2t)$$
$$v(t) = \sqrt{(-\sin t)^2 + (\cos t)^2 + 4t^2} = \sqrt{1+4t^2}$$
$$a(t) = v'(t) = \frac{8t}{2\sqrt{1+4t^2}} = \frac{4t}{\sqrt{1+4t^2}}$$

Svaralternativ (e)

Oppgave 3

$$\vec{r}(t) = (t^2, t^3, t^2) \qquad 0 \le t \le 1$$
$$\vec{v}(t) = (2t, 3t^2, 2t)$$
$$v(t) = \sqrt{(2t)^2 + (3t^2)^2 + (2t)^2} = \sqrt{4t^2 + 9t^4 + 4t^2} = \sqrt{9t^4 + 8t^2}$$
$$= t\sqrt{9t^2+8}$$

$$L(0,1) = \int_0^1 t\sqrt{9t^2+8}\,dt = \int_8^{17} \frac{1}{18}\sqrt{u}\,du = \left[\frac{1}{18}\cdot\frac{2}{3}u^{\frac{3}{2}}\right]_8^{17} = \left[\frac{1}{27}u\sqrt{u}\right]_8^{17}$$

$u = 9t^2+8$

$du = 18t\,dt$

$\frac{1}{18}du = t\,dt$

$$= \frac{1}{27}(17\sqrt{17} - 8\sqrt{8})$$
$$= \frac{1}{27}(17\sqrt{17} - 16\sqrt{2})$$

Svaralternativ (a)

Oppgave 4

$$\vec{F}(x,y,z) = (\underbrace{y+2z}_{\frac{\partial\phi}{\partial x}}, \underbrace{x+3z}_{\frac{\partial\phi}{\partial y}}, \underbrace{2x+3y}_{\frac{\partial\phi}{\partial z}})$$

$$\frac{\partial\phi}{\partial x} = y+2z \Rightarrow \phi(x,y,z) = xy + 2xz + \cancel{C(y,z)}$$

$$\frac{\partial\phi}{\partial y} = x+3z \Rightarrow \phi(x,y,z) = xy + 3yz + D(x,z)$$

$$\frac{\partial\phi}{\partial z} = 2x+3y \Rightarrow \phi(x,y,z) = 2xz + 3yz + \underbrace{E(x,y)}_{xy}$$

$$\Bigg\} \quad xy + 2xz + 3yz$$

$$\phi(x,y,z) = xy + 2xz + 3yz$$

Svaralternativ (c)

Oppgave 5

$$f(x,y) = xe^y + y\cos(\pi x) + 3$$

$$\frac{\partial f}{\partial x} = e^y - \pi y \sin(\pi x)$$

$$\frac{\partial f}{\partial x}(1,0) = 1 - 0 = 1$$

$$\frac{\partial f}{\partial y} = xe^y + \cos(\pi x)$$

$$\frac{\partial f}{\partial y}(1,0) = 1 + \cos(\pi) = 1 - 1 = 0$$

$$f(1,0) = 1 + 0 + 3 = 4$$

Tangentplan i $(1,0)$:

$$f(1,0) + \frac{\partial f}{\partial x}(1,0)(x-1) + \frac{\partial f}{\partial y}(1,0)(y-0)$$

$$= 4 + 1\cdot(x-1) + 0 = 4 + (x-1)$$

Svaralternativ (e)

Oppgave 6

$$\iint_R (x+xy+1)\,dx\,dy = \int_0^2 \left[\int_{-1}^1 (x+xy+1)\,dy\right]dx$$

$$= \int_0^2 \left[xy + \tfrac{1}{2}xy^2 + y\right]_{-1}^{1} dx = \int_0^2 \left(x + \tfrac{1}{2}x + 1 + x - \tfrac{1}{2}x + 1\right)dx$$

$$= \int_0^2 (2x+2)\,dx = \left[x^2 + 2x\right]_0^2 = 4 + 4 = 8$$

Svaralternativ (e)

Oppgave 7

$z = \sqrt{x^2+y^2}$, $z = 2x+1$:

$$\sqrt{x^2+y^2} = 2x+1$$

$$\Downarrow$$

$$x^2+y^2 = (2x+1)^2 = 4x^2+4x+1$$

$$-3x^2 - 4x + y^2 = 1$$

Vi får alltid en hyperbel når det er motsatt fortegn på x^2 og y^2 leddene

Svaralternativ (d)

Oppgave 8

$$\vec{F}(x,y) = \underbrace{\begin{pmatrix} 1 & 4 \\ -6 & 1 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

Rektangelet $R: 1 \le x \le 3$, $2 \le y \le 4$ har areal $2 \cdot 2 = 4$

Fra seksjon 1.10 : $\vec{F}$ forstørrer arealer med faktor $|\det(A)| = 25$

Arealet til $\vec{F}(R)$ blir da $25 \cdot 4 = 100$.

Svaralternativ (a)

Oppgave 9

$A: \; 0 \le y \le \pi$, $0 \le x \le \sin y$

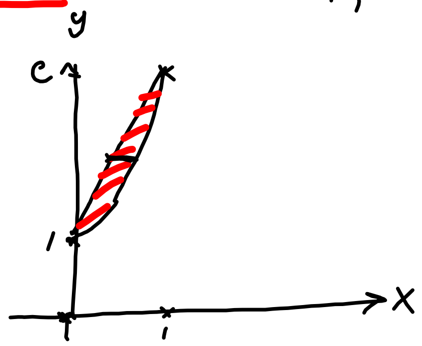
$$\iint_A x\cos y \, dx\, dy = \int_0^\pi \left[\int_0^{\sin y} x \cos y \, dx \right] dy$$

$$= \int_0^\pi \left[\tfrac{1}{2} x^2 \cos y \right]_0^{\sin y} dy = \int_0^\pi \tfrac{1}{2} \underbrace{\sin^2 y}_{\substack{u = \sin y \\ du = \cos y\, dy}} \cos y \, dy$$

$$= \int_0^0 \tfrac{1}{2} u^2 \, du = \left[\tfrac{1}{6} u^3 \right]_0^0 = \underline{0}$$

Svaralternativ (c)

Oppgave 10



$0 \le x \le 1$, $e^x \le y \le (e-1)x+1$

$e^0 = 1$

$e^1 = e$

$e^x = y$

$x = \ln y$

$x=0 \to 1$

$x=1 \to e$

$y = (e-1)x+1$

$y-1 = (e-1)x$

$x = \frac{y-1}{e-1}$

$1 \le y \le e$

x går fra linjen, og opp til e^x, det vil si fra $x = \frac{y-1}{e-1}$, til $x = \ln y$

Integralet: $\int_1^e \left[\int_{\frac{y-1}{e-1}}^{\ln y} f(x,y)\,dx \right] dy$

Svaralternativ (b)

Oppgave 11

$\vec{F}(x,y,z) = (yz, xz, xy)$

$\vec{r}(t) = (\cos t, \sin t, t^2)$ $\frac{\pi}{4} \le t \le \frac{9\pi}{4}$

$\int_C \vec{F} \cdot d\vec{r} = \int_{\frac{\pi}{4}}^{\frac{9\pi}{4}} \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$ gir mye regning

$\phi(x,y,z) = xyz$ er en potensialfunksjon for $\vec{F}$.

Derfor: $\int_C \vec{F} \cdot d\vec{r} = \phi(\vec{r}(\frac{9\pi}{4})) - \phi(\vec{r}(\frac{\pi}{4}))$

(se seksjon 3.5)

$\vec{r}(\frac{\pi}{4}) = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \frac{\pi^2}{16})$

$\vec{r}(\frac{9\pi}{4}) = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \frac{81\pi^2}{16})$

$\phi(\vec{r}(\frac{9\pi}{4})) - \phi(\vec{r}(\frac{\pi}{4})) = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} \cdot \frac{81\pi^2}{16} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} \cdot \frac{\pi^2}{16}$

$= \frac{1}{2} \frac{81\pi^2 - \pi^2}{16} = \frac{80\pi^2}{2 \cdot 16} = \frac{5\pi^2}{2}$

Svaralternativ (c).

Oppgave 12 Arealet til en flate: Se seksjon 6.4

Vi må regne ut $\sqrt{1+\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2} = \sqrt{1+1^2+(\sqrt{2})^2} = 2$

$\left(f(x,y) = x+\sqrt{2}y\right)$

$$\text{Areal} = \int_0^1 \left[\int_1^2 \sqrt{1+\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}\,dy\right]dx$$

$$= 2\int_0^1\left[\int_1^2 1\,dy\right]dx = 2\cdot 1\cdot 1 = 2$$

Svaralternativ (b)

Oppgave 13 $3x^2-12x+2y^2-4y+8=0$

$$3(x^2-4x+4\quad)+2(y^2-2y+1\quad) = -8+12+2$$

$$3(x-2)^2+2(y-1)^2 = 6$$

$$\frac{(x-2)^2}{2}+\frac{(y-1)^2}{3} = 1$$

$$\frac{(x-2)^2}{(\sqrt{2})^2}+\frac{(y-1)^2}{(\sqrt{3})^2} = 1$$

Ellipse med sentrum $(2,1)$

store halvakse $\sqrt{3}$, lille halvakse $\sqrt{2}$.

brennvidde: $c = \sqrt{(\sqrt{3})^2-(\sqrt{2})^2} = \sqrt{3-2} = 1$

brennpunkter: $(2,1) \pm (0,1)$. Dette gir $(2,0)$ og $(2,2)$.

Svaralternativ (c).

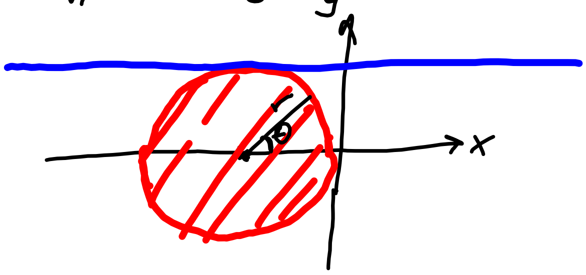
<u>Oppgave 14</u> $x^2+2x+y^2=0$

$x^2+2x+1+y^2=1$

$(x+1)^2+y^2=1$

Sirkel med sentrum $(-1,0)$, radius 1.

Vi har også planet $z=1-y$



(sett ovenfra, $z=0$)

Translaterte polarkoordinater:

$x=r\cos\theta-1$

$y=r\sin\theta$

$$J=\begin{vmatrix}\frac{\partial x}{\partial r} & \frac{\partial x}{\partial\theta}\\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial\theta}\end{vmatrix}=\begin{vmatrix}\cos\theta & -r\sin\theta\\ \sin\theta & r\cos\theta\end{vmatrix}=r$$

$$I=\int_0^{2\pi}\left[\int_0^1(1-y)r\,dr\right]d\theta=\int_0^{2\pi}\left[\int_0^1(1-r\sin\theta)r\,dr\right]d\theta$$

$$=\int_0^{2\pi}\left[\int_0^1(r-r^2\sin\theta)\,dr\right]d\theta=\int_0^{2\pi}\left[\frac{1}{2}r^2-\frac{1}{3}r^3\sin\theta\right]_0^1 d\theta$$

$$=\int_0^{2\pi}\left(\frac{1}{2}-\frac{1}{3}\sin\theta\right)d\theta=2\pi\cdot\frac{1}{2}=\underline{\underline{\pi}}$$

Svaralternativ (e).

Oppgave 15

$$\int_C \underbrace{(x+2y+\sin^2 x)}_{P(x,y)}\,dx + \underbrace{(2x+y+e^{2y})}_{Q(x,y)}\,dy$$

$$= \iint_A \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

her: $\frac{\partial Q}{\partial x} = 2$, $\frac{\partial P}{\partial y} = 2$ slik at $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0$

Derfor blir integralet 0,
Svaralternativ (c).