Bare still spørsmål på chat eller over mikrofon!

Basis: $\vec{v}_1, \dots, \vec{v}_n$:

Dette er en basis for $\mathbb{R}^n$ hvis enhver vektor $\vec{x}$ i $\mathbb{R}^n$ kan skrives unikt:

$$\vec{x} = a_1 \vec{v}_1 + \dots + a_n \vec{v}_n$$

($a_1, \dots, a_n$ er unike).

Eksempel: $\vec{e}_1, \dots, \vec{e}_n$. (standardbasisen)

$$\vec{x} = x_1 \vec{e}_1 + \dots + x_n \vec{e}_n.$$

For å vise at $\vec{v}_1, \dots, \vec{v}_n$ er basis:

Radreduser $[\vec{v}_1, \dots, \vec{v}_n]$

Skal da få identitetsmatrisen.

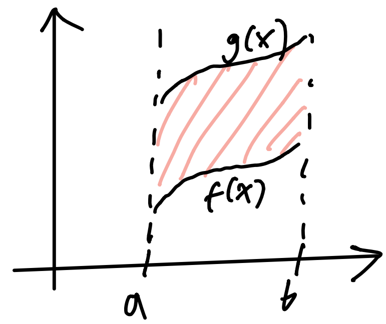
basis: 1) lineær uavhengighet

2) utspenner $\mathbb{R}^n$

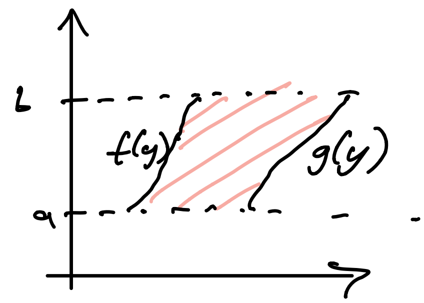
Type I $(a \le x \le b)$

$f(x) \le y \le g(x)$

$$\int_a^b \left[\int_{f(x)}^{g(x)} h(x,y)\,dy \right] dx$$



Type II $(a \le y \le b)$



$f(y) \le x \le g(y)$

$$\int_a^b \left[\int_{f(y)}^{g(y)} h(x,y)\,dx \right] dy$$

Oppgave 6

$$S(x) = \sum_{n=2}^{\infty} \frac{x^n}{n(n-1)4^n} \qquad F(x) = \sum_{n=2}^{\infty} \frac{x^n}{n(n-1)}$$

$$F\left(\frac{x}{4}\right) = S(x)$$

$$S'(x) = \sum_{n=2}^{\infty} \frac{x^{n-1}}{(n-1)4^n}$$

$$S''(x) = \sum_{n=2}^{\infty} \frac{x^{n-2}}{4^n} = \frac{1}{4^2} \sum_{n=2}^{\infty} \frac{x^{n-2}}{4^{n-2}}$$

$u = n-2$

$$= \frac{1}{16} \sum_{u=0}^{\infty} \frac{x^u}{4^u} = \frac{1}{16} \frac{1}{1 - x/4}$$

$(x/4)^u$

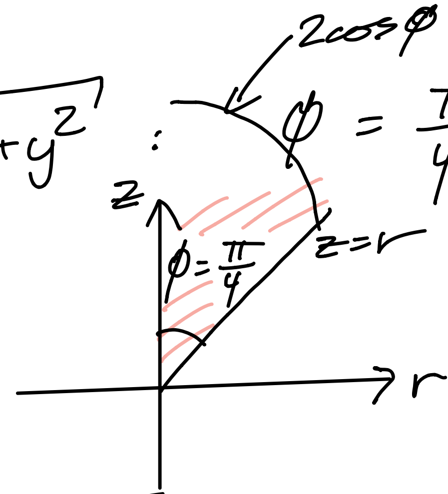
$$F''(x) = \sum_{n=2}^{\infty} x^{n-2} = \frac{1}{1-x}$$

$$F'(x) = -\ln|1-x| + C$$

$$\left(1 - \frac{x}{4}\right)\ln\left(1 - \frac{x}{4}\right) + \frac{x}{4}$$

kjeglen $z = \sqrt{x^2+y^2}$: $\phi = \frac{\pi}{4}$

$z = r$



området: $0 \leq \theta \leq 2\pi$

$0 \leq \phi \leq \frac{\pi}{4}$

$0 \leq \rho \leq 2\cos\phi$

$$V = \int_0^{2\pi}\left[\int_0^{\frac{\pi}{4}}\left[\int_0^{2\cos\phi} \rho^2 \sin\phi \, d\rho\right] d\phi\right] d\theta$$

$\rho = 2\cos\phi \Rightarrow$ kule med radius 1, senter $(0,0,1)$

$\rho^2 = 2\rho\cos\phi$

$x^2+y^2+z^2 = 2z$

$x^2+y^2+z^2-2z+1 = 1$

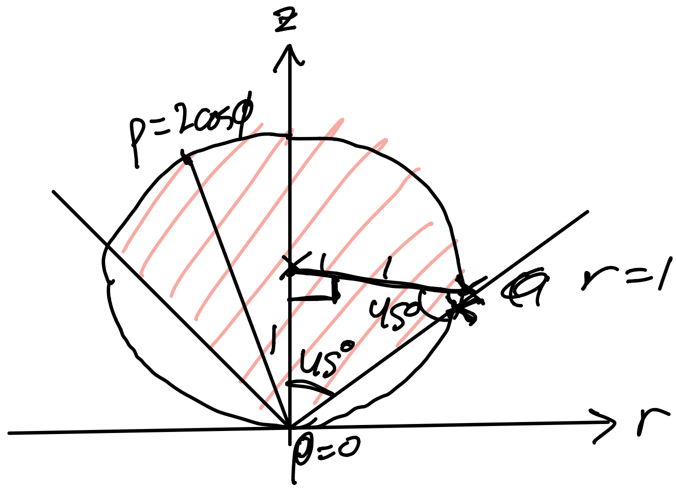
$x^2+y^2+(z-1)^2 = 1$

$r^2+z^2 = 2z$

$2r^2 = 2r$

$r^2 = r$

$r = \pm 1$



$$z = 1 + \sqrt{1-x^2-y^2} \qquad z = 1+\sqrt{1-r^2}$$

sylinderkoordinater:

$$\int_0^{2\pi}\left[\int_0^1\left[\int_r^{1+\sqrt{1-r^2}} r\,dz\right]dr\right]d\theta$$

$$\iiint_V 1\, dx\, dy\, dz$$

$$xyz = V$$

$$z = \frac{V}{xy}$$

$$\nabla f = \lambda \nabla g \qquad g(x,y,z) = xyz - V$$

$$\parallel$$

$$\cdots = \lambda \begin{pmatrix} yz \\ xz \\ xy \end{pmatrix} \begin{array}{l} \cdot x \\ \cdot y \\ \cdot z \end{array}$$

$$= \lambda xyz$$

$$= \lambda xyz$$

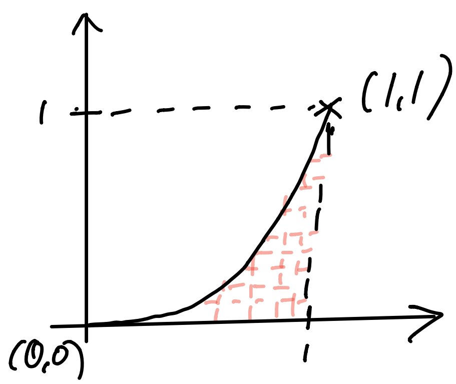
$$= \lambda xyz$$

$$\int_0^1 \left[\int_{y^{\frac{1}{3}}}^1 \frac{\sin(\pi x^2)}{x^2}\, dx \right] dy$$

type II

$x = y^{\frac{1}{3}}$ til $x = 1$

$y = x^3$



~~som~~ type I : $0 \le x \le 1$ $\quad 0 \le y \le x^3$

$$\int_0^1 \left[\int_0^{x^3} \frac{\sin(\pi x^2)}{x^2}\, dy \right] dx$$

$$= \int_0^1 \left[\frac{\sin(\pi x^2)}{x^2}\, y \right]_0^{x^3} dx$$

$$= \int_0^1 x \sin(\pi x^2)\, dx$$

$u = \pi x^2 \quad du = 2\pi x\, dx$

$x\,dx = \frac{1}{2\pi}\, du$

$$= \int_0^{\pi} \frac{1}{2\pi} \sin u\, du = \ldots \left[-\frac{1}{2\pi} \cos u \right]_0^{\pi}$$

$$= \left[-\frac{1}{2\pi}\cos(\pi x^2) \right]_0^1$$

$$A = \int_C x\,dy \qquad P(x,y) = 0$$

$$y'(t)\,dt \qquad Q(x,y) = x$$

$$\underbrace{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}}_{P=0,\ Q=x} = 1$$

$$-\int_C y\,dx$$

$$y(t) = 2\sin t - \sin(2t)$$

$$\int_0^{2\pi} (\underbrace{2\cos t + \cos(2t)}_{x})(2\cos t - 2\cos(2t))\,dt$$

$$-\int_C y\,dx$$

$$S''(x) = \frac{1}{16} \frac{1}{1-\frac{x}{4}}$$

$$S'(x) = \frac{1}{16} \ln\left|1-\frac{x}{4}\right| \cdot (-4) + C$$

$$= -\frac{1}{4} \ln\left|1-\frac{x}{4}\right| + C$$

$S'(0)=0$ (ses fra rekken for $S'(x)$, som ikke har konstantledd).

setter vi inn $x=0$ får vi derfor $C=0$.

$$S'(x) = -\frac{1}{4} \ln\left|1-\frac{x}{4}\right|$$

$$S'(x) = -\frac{1}{4} \cdot \ln\left(1-\frac{x}{4}\right)$$

delvis integrasjon

$$S(x) = -\frac{1}{4} x \ln\left(1-\frac{x}{4}\right) + \frac{1}{4}\int \frac{x}{1-\frac{x}{4}} \left(-\frac{1}{4}\right) dx + C$$

$$= -\frac{1}{4} x \ln\left(1-\frac{x}{4}\right) - \frac{1}{16}\int \frac{x}{1-\frac{x}{4}} dx + C$$

$$\frac{1}{4}\int \frac{x-4+4}{4-x} dx$$

$$\frac{1}{4}\left(\int\left(-1+\frac{4}{4-x}\right)dx\right). + C$$

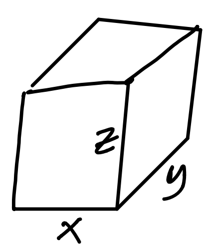
$$S(x) = \cdots + C$$

(sett inn $x=0$, og bruk at $S(0)=0$ for å finne C)

($S(0)=0$ følger fra uttrykket for S som en rekke: denne har ikke noe konstantledd.

Oppgave 6.2.3

Oppgave 7



bunn: xy

sider: $2xz + 2yz$

topp: xy

mål på pris:

$$f(x,y,z) = \underbrace{2xy}_{\text{pris bunn}} + \underbrace{2xz + 2yz}_{\text{pris sider}} + \underbrace{xy}_{\text{pris topp}}$$

$$= 3xy + 2xz + 2yz$$

betingelse: $g(x,y,z) = xyz - V = 0$

To måter å løse på:

1. Lagrange: a) Sjekk om $\nabla g = \vec{0}$

b) $\nabla f = \lambda \nabla g$

2. Substitusjon: sett $z = \frac{V}{xy}$ i $f(x,y,z)$

$$h(x,y) = f\left(x, y, \frac{V}{xy}\right)$$

$$= 3xy + \frac{2V}{y} + \frac{2V}{x}$$

Finn stasjonære punkter, bruk annenderiverttesten

Oppgave 4

$$\frac{R x_n y_n}{5 \cdot 10^6} = \underbrace{R y_n}_{\text{antall som blir smittet når ingen er immune.}} \cdot \underbrace{\frac{x_n}{5 \cdot 10^6}}_{\text{andel av befolkningen som kan smittes.}}$$

ganges disse sammen får vi de som

blir syke ved neste tidssteg (tatt i betraktning at smittbare blir redusert på grunn av immunitet).

$$S(x) = \overset{0}{\left(1 - \frac{x}{4}\right)} \overset{-\infty}{\ln\left(1 - \frac{x}{4}\right)} + \frac{x}{4}$$

$$S(4) = \lim_{x \to 4^-} \underbrace{\underset{\to 0}{\left(1 - \frac{x}{4}\right)} \underset{\to -\infty}{\ln\left(1 - \frac{x}{4}\right)}} + \underset{\to 1}{\frac{x}{4}} = 1$$

$$\lim_{x \to 4} \frac{\ln\left(1 - \frac{x}{4}\right)}{\frac{1}{1 - \frac{x}{4}}} = \lim_{x \to 4} \frac{\frac{1}{1 - \frac{x}{4}}\left(-\frac{1}{4}\right)}{-\frac{1}{\left(1 - \frac{x}{4}\right)^2}\left(-\frac{1}{4}\right)} = \lim_{x \to 4} -\left(1 - \frac{x}{4}\right) = 0$$

$$S(-4) = \lim_{x \to -4^+} \left(1 - \frac{x}{4}\right) \ln\left(1 - \frac{x}{4}\right) + \frac{x}{4}$$

$$= 2 \ln 2 - 1$$

$$S(4) = 1$$

$$S(-4) = 2 \ln 2 - 1$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\dfrac{x^{n+1}}{(n+1)n\,4^{n+1}}}{\dfrac{x^n}{n(n-1)\,4^n}}\right| = \left|\frac{x}{4}\right| \frac{n(n-1)}{(n+1)n} \longrightarrow \left|\frac{x}{4}\right|$$

absolutt konvergens når $\left|\frac{x}{4}\right| < 1$

$$\Updownarrow$$

$$|x| < 4.$$

$x = -4$: $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n(n-1)}$

$x = 4$: $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$

Begge disse konvergerer (sammenlign med $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2}$)

$$\lim_{n\to\infty} \frac{n(n-1)}{(n+1)n} = \lim_{n\to\infty} \frac{n-1}{n+1} = \lim_{n\to\infty} \frac{1-\frac{1}{n}}{1+\frac{1}{n}} = \frac{1}{1} = 1$$