Sek. 1.9 (lineæravbildninger)

Setning 1.9.5 La $T: \mathbb{R}^n \to \mathbb{R}^m$ er en lin. avb., da fins en unik matrise A slik at A en $(m\times n)$-matrise

$$T(\vec{x}) = A\vec{x}$$

Søyle nr. j i A er lik $T(\vec{e}_j) \in \mathbb{R}^m$

Oppg. 1.9.1 $T: \mathbb{R}^3 \to \mathbb{R}^2$ lin. avb. gitt ved

$$T(x,y,z) = \begin{pmatrix} 2x-y+z \\ -x+y-3z \end{pmatrix}$$

Finn A.

Svar: $T(\vec{e}_1) = T(1,0,0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$T(\vec{e}_2) = T(0,1,0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$T(\vec{e}_3) = T(0,0,1) = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$$A = \underline{\underline{\begin{pmatrix} 2 & -1 & 1 \\ -1 & 1 & -3 \end{pmatrix}}}$$

Oppg 1.9.2 $T: \mathbb{R}^2 \to \mathbb{R}^4$ lin. avb. slik at.

$$T(\vec{e}_1) = \begin{pmatrix} -1 \\ 2 \\ -3 \\ 4 \end{pmatrix} \quad T(\vec{e}_2) = \begin{pmatrix} 0 \\ -2 \\ 4 \\ 7 \end{pmatrix}$$

Finn A

Svar (1.9.5) $A = \begin{pmatrix} -1 & 0 \\ 2 & -2 \\ -3 & 4 \\ 4 & 7 \end{pmatrix}$

Oppg. 1.9.3 La $\vec{a}, \vec{b} \in \mathbb{R}^2$. La $T: \mathbb{R}^2 \to \mathbb{R}^2$ være en lin. avb. slik at

$$T(\vec{a}) = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \quad T(\vec{b}) = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

Finn $T(3\vec{a} - 2\vec{b})$.

T lin. avb. $T: \mathbb{R}^n \to \mathbb{R}^m$

Svar: Setning 1.9.2: $T(c_1\vec{x}_1 + c_2\vec{x}_2 + \cdots + c_n\vec{x}_n) = c_1T(\vec{x}_1) + c_2T(\vec{x}_2) + \cdots + c_nT(\vec{x}_n)$

for alle $c_i \in \mathbb{R}$ $\vec{x}_i \in \mathbb{R}^n$

Bruker dette:

$$T(3\vec{a} - 2\vec{b}) = 3T(\vec{a}) - 2T(\vec{b})$$
$$= 3\begin{pmatrix} -2 \\ 1 \end{pmatrix} - 2\begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ -3 \end{pmatrix}.$$

Oppg. 1.9.5 $T: \mathbb{R}^2 \to \mathbb{R}^2$ lin. avb. som fordobler andrekomponent, bevarer førstekomponent. Finn A

Svar: $T(x,y) = \begin{pmatrix} x \\ 2y \end{pmatrix}$

$$T(\vec{e}_1) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$T(\vec{e}_2) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

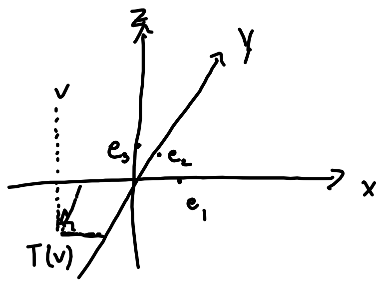
Oppg. 1.9.7 Lin avb $T: \mathbb{R}^3 \to \mathbb{R}^3$ avbilder en vektor på sin projeksjon i xy-planet. Finn A.

Svar: $T(e_1) = e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$T(e_2) = e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$T(e_3) = \vec{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



Seksjon 1.10 (affinavbildninger)

Def 1.10.1 En funksjon $F:\mathbb{R}^n\to\mathbb{R}^m$ er en affinavbildning hvis det fins en $(m\times n)$-matrise A og en vektor $\vec{c}\in\mathbb{R}^m$ slik at

$$F(\vec{x})=A\vec{x}+\vec{c} \quad \text{for alle } \vec{x}\in\mathbb{R}^n.$$

Oppg. 1.10.1 $F:\mathbb{R}^3\to\mathbb{R}^2$ aff. avb. slik at

$$F(x,y,z)=\begin{pmatrix}2x-3y+z-7\\ -x+z-2\end{pmatrix}$$

Finn A & $\vec{c}$.

Svar: $F(\vec{0})=A\vec{0}+\vec{c}=\vec{c}$

$$\vec{c}=F(0,0,0)=\underline{\underline{\begin{pmatrix}-7\\-2\end{pmatrix}}}$$

Søyle nr. j i A er lik $A\vec{e}_j$ $\qquad F(x)=Ax+c$

$$A\vec{e}_j=F(\vec{e}_j)-\vec{c}$$

$$A\vec{e}_1=F(\vec{e}_1)-\vec{c}=F(1,0,0)-\vec{c}=\begin{pmatrix}-5\\-3\end{pmatrix}-\begin{pmatrix}-7\\-2\end{pmatrix}=\begin{pmatrix}2\\-1\end{pmatrix}$$

$$A\vec{e}_2=F(\vec{e}_2)-\vec{c}=\begin{pmatrix}-3\\0\end{pmatrix}$$

$$A\vec{e}_3=\begin{pmatrix}1\\1\end{pmatrix}$$

$$A=\underline{\underline{\begin{pmatrix}2&-3&1\\-1&0&1\end{pmatrix}}}$$

Oppg. 1.10.3 $F:\mathbb{R}^2\to\mathbb{R}^2$ aff. avb. slik at

$$F(0,0)=\begin{pmatrix}1\\-1\end{pmatrix}\quad F(1,0)=\begin{pmatrix}2\\3\end{pmatrix}\quad F(0,1)=\begin{pmatrix}-1\\0\end{pmatrix}$$

Finn A & $\vec{c}$

Svar: $F(x)=Ax+c$

$$c=F(0)=\underline{\underline{\begin{pmatrix}1\\-1\end{pmatrix}}}$$

$$A\vec{e}_1=F(\vec{e}_1)-\vec{c}=\begin{pmatrix}2\\3\end{pmatrix}-\begin{pmatrix}1\\-1\end{pmatrix}=\begin{pmatrix}1\\4\end{pmatrix}$$

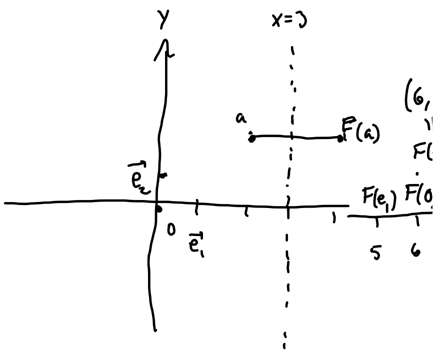
$$A\vec{e}_2=\begin{pmatrix}-2\\1\end{pmatrix}$$

$$A=\underline{\underline{\begin{pmatrix}1&-2\\4&1\end{pmatrix}}}$$

Oppg. 1.10.5a) $F:\mathbb{R}^2\to\mathbb{R}^2$ affinavb. som avbilder hvert punkt på sitt speilbilde om linja $x=3$.

Finn A og $\vec{c}$

Svar:



$$F(\vec{x})=A\vec{x}+\vec{c}$$

$$\vec{c}=F(\vec{0})=\underline{\underline{\begin{pmatrix}6\\0\end{pmatrix}}}$$

$$A\vec{e}_1=F(\vec{e}_1)-\vec{c}=\begin{pmatrix}5\\0\end{pmatrix}-\begin{pmatrix}6\\0\end{pmatrix}=\begin{pmatrix}-1\\0\end{pmatrix}$$

$$A\vec{e}_2=F(\vec{e}_2)-\vec{c}=\begin{pmatrix}6\\1\end{pmatrix}-\begin{pmatrix}6\\0\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}$$

$$A=\underline{\underline{\begin{pmatrix}-1&0\\0&1\end{pmatrix}}}$$

Seksjon 2.7 (kjerneregelen)

Oppg. 2.7.1 La $f(u,v) = u^2 + v$, $g(x,y) = 2xy$, $h(x,y) = x + y^2$.

Finn de partiellderiverte av $k(x,y) = f(g(x,y), h(x,y))$ ved hjelp av kjerneregelen.

Svar: $k(x,y) = g(x,y)^2 + h(x,y) = 4x^2y^2 + x + y^2$

Kjerneregelen gir

$$\frac{\partial k}{\partial x}(x,y) = \frac{\partial f}{\partial u}(g(x,y), h(x,y)) \cdot \frac{\partial g}{\partial x}(x,y) + \frac{\partial f}{\partial v}(g(x,y), h(x,y)) \cdot \frac{\partial h}{\partial x}(x,y)$$

$$\frac{\partial f}{\partial u} = 2u \qquad \frac{\partial f}{\partial v} = 1$$

$$\frac{\partial g}{\partial x} = 2y \qquad \frac{\partial h}{\partial x} = 1 \qquad \frac{\partial g}{\partial y} = 2x \qquad \frac{\partial h}{\partial y} = 2y$$

$$\frac{\partial k}{\partial x}(x,y) = 2u \cdot 2y + 1 \cdot 1 \qquad (u = g(x,y))$$

$$= 2 \cdot 2xy \cdot 2y + 1 = \underline{\underline{8xy^2 + 1}}.$$

$$\frac{\partial k}{\partial y}(x,y) = \frac{\partial f}{\partial u}(g(x,y), h(x,y)) \cdot \frac{\partial g}{\partial y}(x,y)$$
$$+ \frac{\partial f}{\partial v}(g(x,y), h(x,y)) \cdot \frac{\partial h}{\partial y}(x,y)$$

$$= 2u \cdot 2x + 1 \cdot 2y = 2 \cdot 2xy \cdot 2x + 2y$$
$$= \underline{\underline{8x^2y + 2y}}.$$

Oppg. 2.7.5 $G: \mathbb{R}^2 \to \mathbb{R}^3$ & $F: \mathbb{R}^3 \to \mathbb{R}^2$ er deriverbare funksjoner. Anta $G(1,-2) = (1,2,3)$ og

$$G'(1,-2) = \begin{pmatrix} 1 & -2 \\ 3 & 1 \\ 2 & -1 \end{pmatrix} \qquad F'(1,2,3) = \begin{pmatrix} 2 & 1 & 4 \\ 0 & 2 & 2 \end{pmatrix}$$

La $H(\vec{x}) = F(G(\vec{x}))$, finn Jacobi-matrisen $H'(1,-2)$.

Svar: Kjerneregelen på matriseform sier

$$H'(\vec{x}) = F'(G(\vec{x}))\, G'(\vec{x})$$

Sett $\vec{x} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$$H'(1,-2) = F'(G(1,-2))\, G'(1,-2)$$
$$= F'(1,2,3)\, G'(1,-2)$$
$$= \begin{pmatrix} 2 & 1 & 4 \\ 0 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 3 & 1 \\ 2 & -1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 13 & -7 \\ 10 & 0 \end{pmatrix}}}.$$

Seksjon 2.8 (Linearisering)

Def. 2.8.2 Gitt en funksjon $F:\mathbb{R}^n \to \mathbb{R}^m$, deriverbar i $\vec{a} \in \mathbb{R}^n$, så er lineariseringen til F i $\vec{a}$ affinavbildningen $T_{\vec{a}}F:\mathbb{R}^n \to \mathbb{R}^m$ gitt ved

$$T_{\vec{a}}F(\vec{x}) = F(\vec{a}) + F'(\vec{a})\cdot(\vec{x}-\vec{a}).$$

Oppg. 2.8.2. La $F:\mathbb{R}^2 \to \mathbb{R}^3$ være gitt ved

$$F(x,y) = \begin{pmatrix} x\sin(xy) \\ xe^y \\ 2x^3+y \end{pmatrix}$$

Finn $T_{\vec{a}}F$ for $\vec{a} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

Svar: $F(\vec{a}) = F(2,0) = \begin{pmatrix} 2\sin(2\cdot 0) \\ 2e^0 \\ 2\cdot 2^3+0 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 16 \end{pmatrix}$

$$F'(\vec{x}) = \begin{pmatrix} \sin(xy)+x\cdot y\cos(xy) & x^2\cos(xy) \\ e^y & xe^y \\ 6x^2 & 1 \end{pmatrix}$$

$$F'(2,0) = \begin{pmatrix} \sin(0)+2\cdot 0\cdot\cos(0) & 2^2\cos(0) \\ e^0 & 2e^0 \\ 6\cdot 2^2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 4 \\ 1 & 2 \\ 24 & 1 \end{pmatrix}$$

$$T_{\vec{a}}F = F(\vec{a}) + F'(\vec{a})(\vec{x}-\vec{a})$$

$$= \begin{pmatrix} 0 \\ 2 \\ 16 \end{pmatrix} + \begin{pmatrix} 0 & 4 \\ 1 & 2 \\ 24 & 1 \end{pmatrix}\left(\vec{x} - \begin{pmatrix} 2 \\ 0 \end{pmatrix}\right)$$

$$= \begin{pmatrix} 0 & 4 \\ 1 & 2 \\ 24 & 1 \end{pmatrix}\vec{x} + \begin{pmatrix} 0 \\ 2 \\ 16 \end{pmatrix} - \begin{pmatrix} 0 & 4 \\ 1 & 2 \\ 24 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 4 \\ 1 & 2 \\ 24 & 1 \end{pmatrix}\vec{x} + \begin{pmatrix} 0 \\ 2 \\ 16 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 48 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} 0 & 4 \\ 1 & 2 \\ 24 & 1 \end{pmatrix}}_{A}\vec{x} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ -32 \end{pmatrix}}_{\vec{c}}$$