3.1 1, 2, 3, 5ab, 8, 10, 14, 21

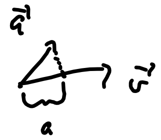
1. $\vec{r}(t) = (t^3, t^2)$

$\vec{v}(t) = \frac{d}{dt}(\vec{r}(t)) = (3t^2, 2t)$

$v(t) = |\vec{v}(t)| = \sqrt{(3t^2)^2 + (2t)^2} = \sqrt{9t^4 + 4t^2}$

$\vec{a}(t) = \frac{d}{dt}\vec{v}(t) = (6t, 2)$

$\frac{d}{dt}v(t) \overset{?}{=} a(t) = \frac{\vec{v}}{|v|} \cdot \vec{a} = \frac{1}{\sqrt{9t^4+4t^2}}(3t^2, 2t)\cdot(6t, 2)$



$= \frac{1}{\sqrt{9t^4+4t^2}}(18t^3 + 4t) = \underline{\frac{18t^2+4}{\sqrt{9t^2+4}}}$

2) $\vec{r}(t) = (\cos t, t\sin t)$

$\vec{v}(t) = (-\sin t, \sin t + t\cos t)$

$v(t) = \sqrt{\sin^2 t + \sin^2 t + 2t\sin t\cos t + t^2\cos^2 t} = \sqrt{2\sin^2 t + t\sin 2t + t^2\cos^2 t}$

$\vec{a}(t) = (-\cos t, \cos t + \cos t - t\sin t) = (-\cos t, 2\cos t - t\sin t)$

$a(t) = \frac{1}{\sqrt{}}(-\sin t, \sin t + t\cos t)\cdot(-\cos t, 2\cos t - t\sin t)$

$= \frac{1}{\sqrt{}}\big(\sin t\cos t + 2\sin t\cos t + t(2\cos^2 t - \sin^2 t) - t^2\sin t\cos t$

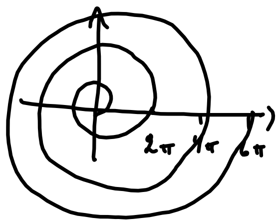
$= \frac{1}{\sqrt{}}\left(\frac{3}{2}\sin 2t + t(\cos 2t + \cos t) - \frac{1}{2}t^2\sin 2t\right)$

3 $\vec{r}(t) = (t, e^{-t}, \sin t)$

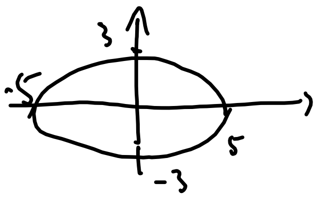
$\vec{v}(t) = (1, -e^{-t}, \cos t)$

$\vec{a}(t) = (0, e^{-t}, -\sin t)$

5 a a $\vec{r}(t) = (t\cos t, t\sin t)$ $t \in [0, 6\pi]$



b) $\vec{r}(t) = (5\cos t, 3\sin t)$ $t \in [0, 2\pi]$



8 $\vec{r}(t^2, t^3)$ $t \in [0, 10]$

$$L = \int ds$$

$$= \int_0^{10} v(t)\, dt$$

$$= \int_0^{10} \sqrt{4t^2 + 9t^4}\, dt = \int_0^{10} t\sqrt{4 + 9t^2}\, dt$$

$$= \int_0^{10} \frac{18t}{18} \sqrt{4 + 9t^2}\, dt = \frac{1}{18} \Big| _0^{10} \frac{2}{3}(4 + 9t^2)^{\frac{3}{2}} = \frac{1}{27}\left(904^{3/2} - 8\right)$$

$$= \frac{904^{3/2} - 8}{27}$$

$$ds = |\vec{r}\,'(t)|\, dt$$
$$= |\vec{v}(t)|\, dt$$
$$= v(t)\, dt$$
$$= |(2t, 3t^2)|\, dt = \sqrt{4t^2 + 9t^4}\, dt$$

10 $\vec{r}(t) = (2\cos t, \sqrt{2}\sin t, \sqrt{2}\sin t)$

a) $\vec{r}'(t) = (-2\sin t, \sqrt{2}\cos t, \sqrt{2}\cos t)$

$v(t) = \sqrt{4\sin^2 t + 2\cos^2 t + 2\cos^2 t} = 2\sqrt{\sin^2 t + \cos^2 t} = \underline{\underline{2}}$

$\vec{a}(t) = (-2\cos t, -\sqrt{2}\sin t, -\sqrt{2}\sin t) = -\vec{r}(t)$

b) $L = \int ds = \int_0^{2\pi} v\, dt = \int_0^{2\pi} 2\, dt = \Big|_0^{2\pi} 2t = \underline{\underline{4\pi}}$ $t \in [0, 2\pi]$

c) kuleflate med radius a og sentrum i $(0,0,0)$: $x^2 + y^2 + z^2 = a^2$

$(2\cos t)^2 + (\sqrt{2}\sin t)^2 + (\sqrt{2}\sin t)^2 =$

$= 4\cos^2 t + 2\sin^2 t + 2\sin^2 t = \underline{4}$

$\Rightarrow$ kurven ligger på kuleflate med radius $\underline{2}$.

d) $\sqrt{2}\sin t - \sqrt{2}\sin t = 0$ for alle t

Så ligger kurven i planet $y - z = 0$.

e) kurven er en sirkel med radius 2.

14 $\vec{r}(t) = (r\, x(t), \; r\, y(t))$

$v = \text{constant} = v.$

$x(t) = \cos(\alpha(t)) \qquad y(t) = \sin(\alpha(t))$

$\vec{v}(t) = r(-\sin(\alpha(t)) \cdot \alpha'(t), \; \cos(\alpha(t))\, \alpha'(t))$

$v = v(t) = r\sqrt{(\alpha'(t))^2 (\sin^2(\alpha(t)) + \cos^2(\alpha(t)))} = r\alpha'(t)$

$\alpha'(t) = \frac{v}{r} \quad \Rightarrow \quad \alpha(t) = \frac{v}{r} t + c$

$\alpha(0) = 0 \quad \Rightarrow \quad c = 0$

$\vec{r}(t) = (r \cos \frac{v}{r} t, \; r \sin \frac{v}{r} t)$

b) $\vec{v}(t) = (r \cdot \frac{v}{r}(-\sin \frac{vt}{r}), \; r \frac{v}{r} \cos \frac{v}{r} t)$

$\vec{a}(t) = (-\frac{v \cdot v}{r} \cos \frac{vt}{r}, \; -\frac{v^2}{r} \sin \frac{v}{r} t) = -\frac{v^2}{r^2} \vec{r}(t)$

$= \underline{-(\frac{v}{r})^2 \vec{r}(t)}.$

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$$\vec{a}(t) = k(t)\,\vec{r}(t) \qquad \vec{r}(t) = (x(t), y(t), z(t))$$
$$\vec{v}(t) = (x'(t), y'(t), z'(t))$$

a) $\frac{d}{dt}[\vec{r}(t) \times \vec{v}(t)]$

$$= \frac{d}{dt}\left(y(t)z'(t) - y'(t)z(t),\ -x(t)z'(t) + x'(t)z(t),\ x(t)y'(t) - x'(t)y(t)\right)$$

$y = y(t)$ etc

$$= (y'z' + yz'' - y''z - y'z',\ -x'z' - xz'' + x''z + x'z',\ x'y' + xy'' - x''y - x'y')$$

$$= (yz'' - y''z,\ -xz'' + x''z,\ xy'' - x''y)$$

$$= \vec{r}(t) \times \vec{a}(t) = \vec{r}(t) \times (k(t)\cdot\vec{r}(t)) = k(t)(\vec{r}(t) \times \vec{r}(t))$$

$$= \underline{0}$$

b) a) $\Rightarrow \frac{d}{dt}(\vec{r}(t) \times \vec{v}(t)) = 0 \Rightarrow \vec{r}(t) \times \vec{v}(t) = \vec{c}$

c) $\vec{0}$, $\vec{r}(0)$, $\vec{v}(0)$

$\vec{r}(t) \times \vec{v}(t) = \vec{c}$ er konstant $\qquad \vec{c} = (\alpha, \beta, \gamma)$

$\Rightarrow$ $\vec{c}$ er normal til $\vec{r}$ og $\vec{v}$ for alle t.

spesielt for $\vec{r}(0)$ og $\vec{v}(0)$, så

$\vec{r}$ og $\vec{v}$ er alltid i planet med normal vektor $\vec{c}$.

I tillegg er $\vec{0} \cdot \vec{c} = 0$ ~~[illegible]~~, så

partikkelbanen er alltid i planet utspent av $\vec{0}$, $\vec{r}(0)$ og $\vec{v}(0)$.

$\underline{3.2}$ 1, 3, 5, 7

1) $f(x,y) = x^2y^3 \qquad \vec{r}(t) = (t^2, 3t)$

$g(t) = f(\vec{r}(t)) = (t^2)^2 \cdot (3t)^3 = 27t^7$

$g'(t) = \underline{7 \cdot 27 \cdot t^6} = \underline{189t^6}$

3) $f(x,y,z) = x^2z - y\sin(yz) \qquad \vec{r}(t) = (e^t, t, \cos t^2)$

$g(t) = f(\vec{r}(t)) = e^{2t}\cos t^2 - t\sin t(\cos t^2)$

$g'(t) = 2e^{2t}\cos t^2 + e^{2t}(-\sin t^2)2t - \sin t(\cos t^2)$

$- t\cos(t\cos(t^2)) \cdot (\cos(t^2) - t\sin t^2 \cdot 2t)$

$= \underline{2e^{2t}\cos t^2} - 2te^{2t}\sin t^2 - \sin(t(\cos(t^2)))$

$- t\cos(t\cos(t^2)) \cdot \underline{(\cos t^2 - 2t^2\sin t^2)}$

5 $\vec{F}(x,y) = \begin{pmatrix} x^2 y \\ xy + x \end{pmatrix}$ $\quad \vec{r}(t) = (\sin t, \cos t)$

$$\vec{G}(t) = \vec{F}(\vec{r}(t)) = \begin{pmatrix} \sin^2 t \cos t \\ \sin t \cos t + \sin t \end{pmatrix}$$

$$\vec{G}'(t) = \begin{pmatrix} 2\sin t \cos^2 t + \sin^2 t(-\sin t) \\ \cos^2 t - \sin^2 t + \cos t \end{pmatrix} = \underline{\begin{pmatrix} 2\sin t \cos^2 t - \sin^3 t \\ \cos 2t + \cos t \end{pmatrix}}$$

7 $f(x,y,t) = 20 + 2t - x^2 + y^2$

$$\vec{r}(t) = \left(3t - \frac{t^2}{4},\ 2t + \frac{t^2}{8}\right)$$

$$g(t) = f(\vec{r}(t)) = 20 + 2t - \left(3t - \frac{t^2}{4}\right)^2 + \left(2t + \frac{t^2}{8}\right)^2$$

$$= 20 + 2t - 9t^2 + \frac{3}{2}t^3 - \frac{1}{16}t^4 + 4t^2 + \frac{1}{2}t^3 + \frac{1}{64}t^4$$

$$g'(t) = 2 - 18t + \frac{9}{4}t^2 - \frac{1}{4}t^3 + 8t + \frac{3}{2}t^2 + \frac{1}{16}t^3$$

$$g'(1) = 2 - 18 + \frac{9}{4} - \frac{1}{4} + 8 + \frac{3}{2} + \frac{1}{16} = -8 - \frac{3}{16} < 0$$

temperaturen avtar.

3.3 1. $\int_C f\,ds \qquad f(x,y) = x \qquad C: \vec{r}(t) = (\sin t, \cos t)$

$t \in [0, 2\pi]$

$= \int_0^{2\pi} \sin t \cdot 1 \cdot dt = 0$

$ds = v(t)\,dt$

$v(t) = |r'(t)| = \sqrt{(\cos t)^2 + (-\sin t)^2}$

$= \underline{1}$

$\underline{4}$ $\int_C f\,ds \qquad f(x,y,z) = xz \qquad C: \vec{r}(t) = (2t^3, 3\sqrt{2}t^2, 6t) \quad t \in [0,1]$

$ds = v(t)\,dt$

$= \sqrt{(6t^2)^2 + (3\sqrt{2}\cdot 2t)^2 + 6^2}\,dt$

$= \sqrt{(36t^4 + 72t^2 + 36)}\,dt$

$= 6\,(t^2+1)\,dt$

$= \int_0^1 2t^3 \cdot 6t\,(6(t^2+1))\,dt$

$= 72\int_0^1 t^6 + t^4\,dt = 72\left|_0^1 \left(\frac{1}{7}t^7 + \frac{1}{5}t^5\right)\right.$

$= 72\left(\frac{1}{7} + \frac{1}{5}\right) = \frac{72}{35}(5+7) = \underline{\frac{12 \cdot 72}{35}}$ D.