Forelesning, 2. februar 2021

tirsdag 2. februar 2021 12:09

Grafisk framstilling av
- skalarfelt
- vektorfelt.

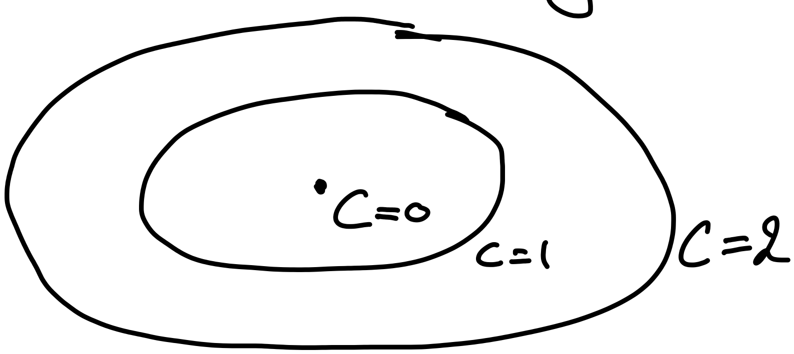
Eks. $f: \mathbb{R}^2 \to \mathbb{R}$ $f(x,y) = x^2 + 4y^2$

Nivåkurver $N_c = \{(x,y) \in \mathbb{R}^2 \mid f(x,y) = c\}$

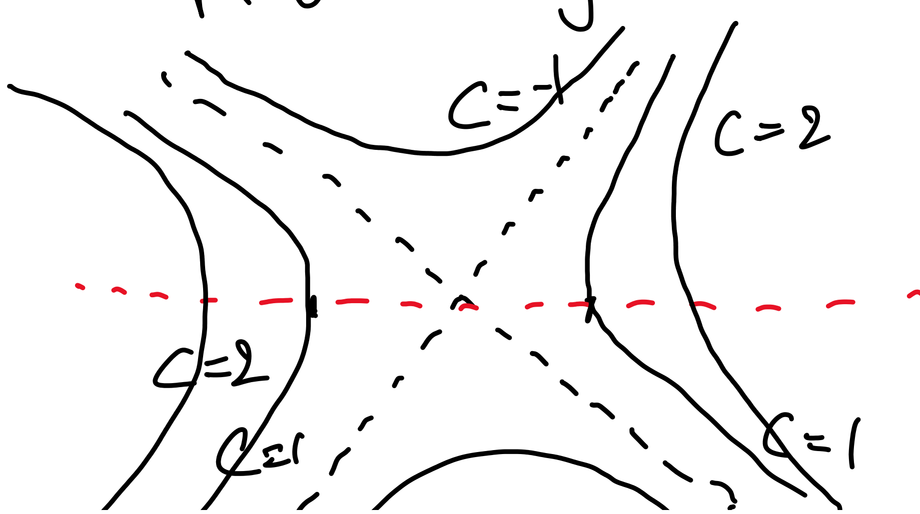
i Eks $x^2 + 4y^2 = c$

$N_0: x^2 + 4y^2 = 0$

(For $c < 0$: $N_c = \emptyset$)

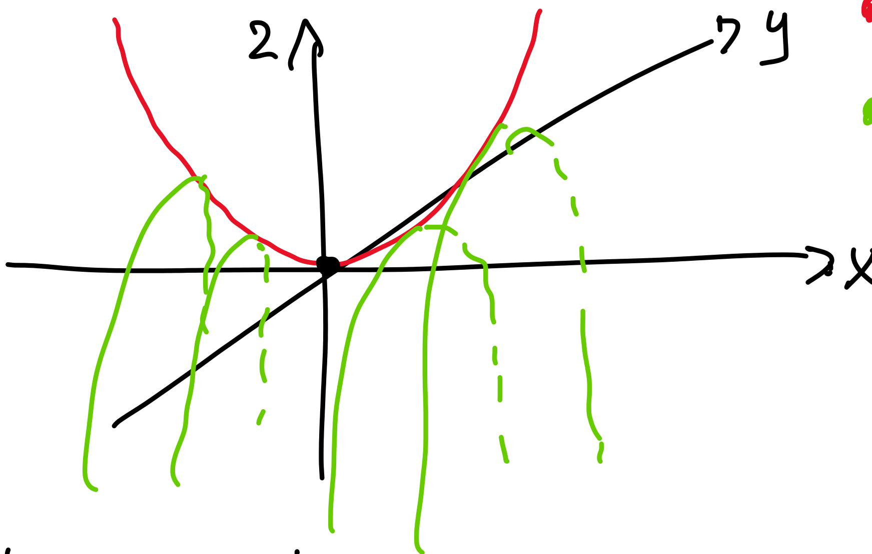


Eks. $z = f(x,y) = x^2 - y^2 = c$



Nivåkurver for $f(x,y)$

3-dim tegning $z = x^2 - y^2$



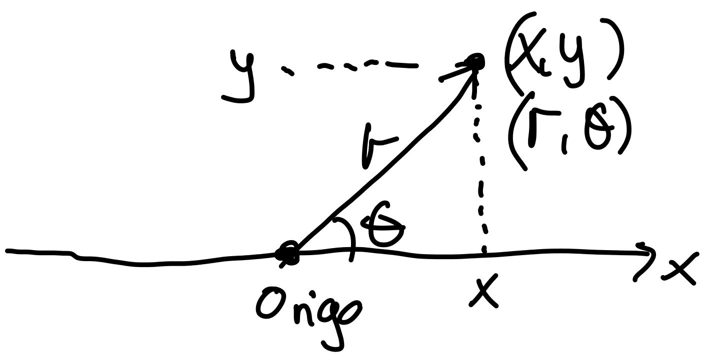
$z = x^2$

$x = 1, z = 1 - y^2$

$x = 2, z = 4 - y^2$

$\vdots$

Polarkoordinater



Kartesisk : (x, y)
Rettvinklede koordinater

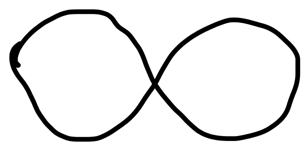
Polarkoordinater
(r, θ)

Overgangen : $x = r\cos\theta$
$y = r\sin\theta$

eller
$$r = \sqrt{x^2 + y^2}$$
$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}, \quad \sin\theta = \frac{y}{\sqrt{x^2+y^2}}$$

Eks.

$$\bullet\ (x^2+y^2)^2 = a^2(x^2-y^2)$$



Lemniskate

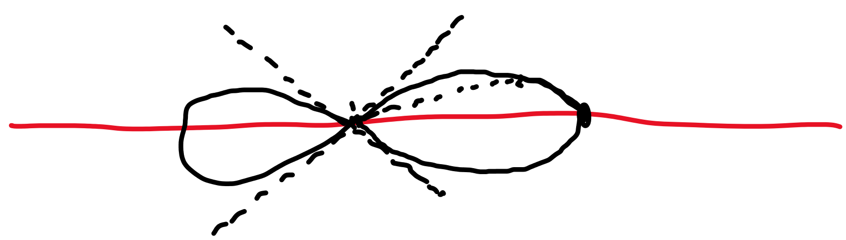
$$\bullet\ x = \frac{a\cdot\cos t}{1+\sin^2 t},\quad y = \frac{a\sin t\cos t}{1+\sin^2 t}$$

$$\left((r\cos\theta)^2+(r\sin\theta)^2\right)^2 = a^2\left((r\cos\theta)^2-(r\sin\theta)^2\right)$$

$$\left(r^2(\underbrace{\cos^2\theta+\sin^2\theta}_{=1})\right)^2 = a^2r^2(\underbrace{\cos^2\theta-\sin^2\theta}_{\cos 2\theta}) \quad \Big|\ \frac{1}{r^2}$$

$$\bullet\ r^2 = a^2\cos 2\theta$$

$(a=1)$



$\theta=0 \quad r^2=a^2$

$\theta=\frac{\pi}{4} \quad r=0$

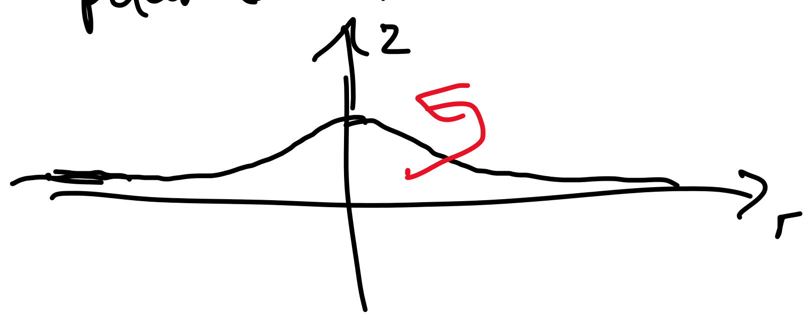
$\theta=\frac{\pi}{2} \quad r^2=-a^2$

$\theta=\frac{3\pi}{4} \quad r=0$

$\theta=\pi \quad r^2=a^2$

Eks. $z=f(x,y) = e^{-(x^2+y^2)}$

i polarkoordinater $z=e^{-r^2}$



I rommet :

En hatt. (drei kurven rundt)

Funksjoner i 3 variable

Funksjoner i 3 variable

Nivåflater: $N_c = \{(x,y,z) \in \mathbb{R}^3 \mid f(x,y,z) = c\}$

Eks $f(x,y,z) = x^2 + y^2 + z^2$

$$N_1 = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$$

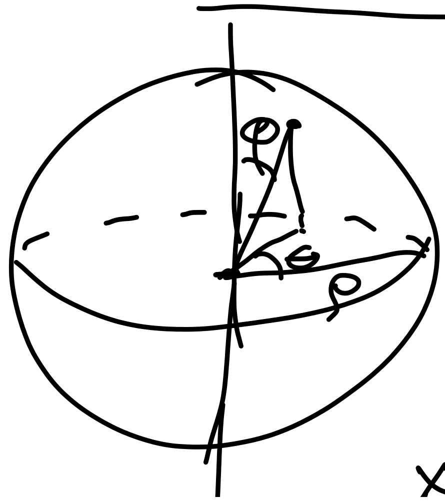
Kuleskall med radius = 1

Eks. Temperatur: $T(x,y,z) =$ temperatur i punktet (x,y,z)

Nivåflater: Isotermer

lik   temperatur

Kulekoordinater / sfæriske koordinater



ρ: avstand fra origo

θ: vinkelen i xy-planet med x-aksen

φ: vinkelen med positiv z-akse

$x = \rho \cdot \cos\theta \cdot \sin\varphi$

$f(x,y,z) = x^2+y^2+z^2$ $\qquad y = \rho \cdot \sin\theta \cdot \sin\varphi$

$\qquad z = \rho \cdot \cos\varphi$

Eks. 1) Kuleskall : ρ = radius f.eks $\underline{\rho = 1}$

2) $u = f(x,y,z) = x^2+y^2-z^2 \rightsquigarrow \underline{u = -\rho^2 \cos 2\varphi}$

Nivåflate, tangentplan, normalvektorer

Setn. 3.7.8

$f : A \to \mathbb{R}$, $A \subset \mathbb{R}^n$, $a \in A$ $\qquad \nabla f(\underline{a}) \perp N_c$

$f(\underline{a}) = c$

som betyr: $\bar{r}(t)$ kurve på N_c

$f(\bar{r}(t)) = c$

$\nabla f(\underline{a}) \cdot \bar{r}'(t) = 0$

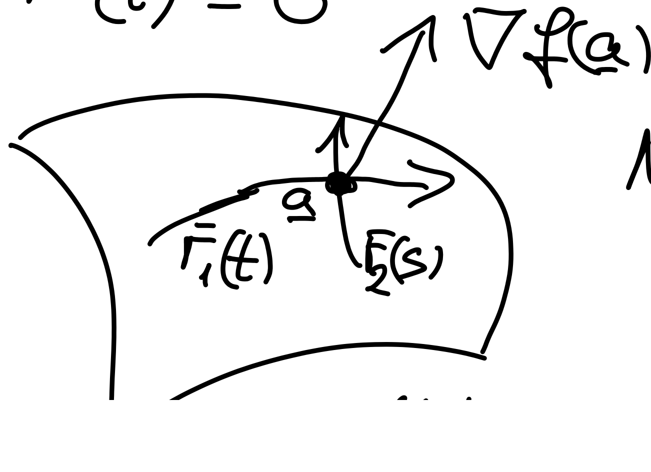
$\underline{a} = \bar{r}(t)$

Bevis.

$f(\bar{r}(t)) = c$ konstant

derivere mhp t :

$\nabla f(\underline{a}) \cdot \bar{r}'(t) = 0$



$\underline{a} = \bar{r}_1(t_0)$

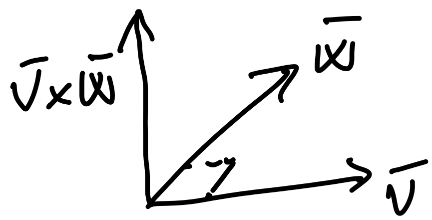
$\underline{a} = \bar{r}_2(s_0)$

Normalvektor til flaten

$\bar r_1'(t_0)\cdot \bar r_2'(s_0)=0$ / Nivåflate N_c

står normalt på hverandre

Normalvektor: $\bar n(\underline{a}) = \bar r_1'(t_0)\times \bar r_2'(s_0)$



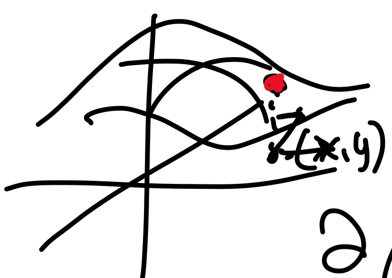
Foregripe parametriserte flater

$$\bar r(s,t) = \big(x(s,t),\ y(s,t),\ z(s,t)\big)$$

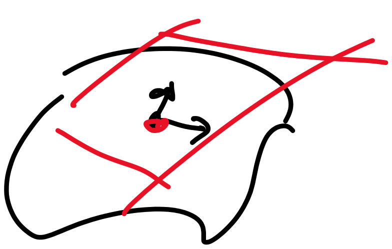
$$\frac{\partial}{\partial s}\big(\bar r(s,t)\big) = \bar r_s(s,t)$$

$$\frac{\partial}{\partial t}\big(\bar r(s,t)\big) = \bar r_t(s,t)$$

Funksjon $z=f(x,y)$ Parametriser ved



$$\bar r(x,y) = \big(x,\ y,\ f(x,y)\big)$$



$$\frac{\partial}{\partial x}\big(\bar r(x,y)\big) = \left(1,\ 0,\ \frac{\partial f}{\partial x}\right) = \bar r_x$$

$$\frac{\partial}{\partial y}\big(\bar r(x,y)\big) = \left(0,\ 1,\ \frac{\partial f}{\partial y}\right) = \bar r_y$$

normalvektor: $\bar r_x \times \bar r_y = \begin{vmatrix} i & j & k \\ 1 & 0 & \frac{\partial f}{\partial x} \end{vmatrix}$

$$\begin{vmatrix} 0 & 1 & \frac{\partial f}{\partial y} \end{vmatrix}$$

$$= \left(-\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 1\right)$$

Tangent planet til flaten i $\underline{a}$:

Et plan gjennom $\underline{a}$ som akkurat tangerer flaten:

Finner som et plan som står normalt på normalvektoren

$$\bar{n} \cdot (\bar{x} - \bar{a}) = 0$$

$$\frac{\partial f}{\partial x}(a)(x - a_1) + \frac{\partial f}{\partial y}(a)(y - a_2) + 1(z - a_3) = 0$$

Likningen til et plan

Eks

$2: f(x,y) = x^3 y^2 \quad \underline{a} = (2,-1,8)$

$$\frac{\partial f}{\partial x} = 3x^2y^2 \quad \frac{\partial f}{\partial x}(2,-1) = 12$$

$$\frac{\partial f}{\partial y} = 2x^3y \quad \frac{\partial f}{\partial y}(2,-1) = -16$$

$$\underline{n} = (-12, 16, 1)$$

Tangentplanet:

$$(-12, 16, 1) \cdot (x-2, y+1, z-8) = 0$$

$$-12x + 24 + 16y + 16 + z - 8 = 0$$

$$\boxed{-12x + 16y + z = -32}$$