# Forelesning, 15. mars 2021

mandag 15. mars 2021 14:18

2019

3. $v_1 = \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix}$ $v_2 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ $w = \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix}$

$$x \cdot \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix} + y \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2x+y \\ 3x \\ 7x+2y \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix} \leftarrow x = -\frac{2}{3}$$

setter $x = -\frac{2}{3}$ inn i 1. og 3. rad:

$$\left.\begin{aligned} 2\left(-\tfrac{2}{3}\right) + y &= 2 \\ 7\left(-\tfrac{2}{3}\right) + 2y &= 2 \end{aligned}\right| \cdot 3$$

$$-4 + 3y = 6 \Rightarrow 3y = 10$$
$$-14 + 6y = 6 \Rightarrow 6y = 20$$

går ~~ikke~~

$$\Rightarrow \quad y = \frac{10}{3} \quad \text{dvs } w = -\frac{2}{3} v_1 + \frac{10}{3} v_2$$

Alternativ 1

4. $A = \begin{pmatrix} 1 & 0 & 4 \\ 2 & 2 & 3 \\ 1 & 2 & -1 \end{pmatrix}$ $Ax = 0$, $Ax = b$, Trappeform, det.

$$\xrightarrow{l_2 \to l_2 - 2l_1} \begin{pmatrix} 1 & 0 & 4 \\ 0 & 2 & -5 \\ 1 & 2 & -1 \end{pmatrix} \xrightarrow{l_3 \to l_3 - l_1} \begin{pmatrix} 1 & 0 & 4 \\ 0 & 2 & -5 \\ 0 & 2 & -5 \end{pmatrix} \xrightarrow[l_2 \to \frac{1}{2} l_2]{l_3 \to l_3 - l_2} \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & -\frac{5}{2} \\ 0 & 0 & 0 \end{pmatrix}$$

$\det(A) = 0 \Leftarrow \det = 0$

5. $\begin{pmatrix} 3 & 4 \\ 0 & 2 \end{pmatrix}$ øvre triangulær matrise

$\Rightarrow$ egenverdier: 5, 2

9. $A = \begin{pmatrix} 5 & -1 \\ -1 & 5 \end{pmatrix}$ $\quad \det \begin{pmatrix} 5-\lambda & -1 \\ -1 & 5-\lambda \end{pmatrix} = \lambda^2 - 10\lambda + 25 - 1$

symmetrisk $\Rightarrow$ ortonormal basis av egenvektorer

$$= \lambda^2 - 10\lambda + 24 = 0$$

abc : $\lambda = \frac{10 \pm \sqrt{100-96}}{2} = \frac{10 \pm 2}{2} = \begin{cases} 6 \\ 4 \end{cases}$

$\underline{\lambda_1 = 6}$ $\quad \begin{pmatrix} 5-6 & -1 \\ -1 & 5-6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x-y \\ -x-y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

dvs $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -x \end{pmatrix} = x \underline{\underline{\begin{pmatrix} 1 \\ -1 \end{pmatrix}}}$

$\underline{\lambda_2 = 4}$ $\quad \begin{pmatrix} 5-4 & -1 \\ -1 & 5-4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x-y \\ x-y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

dvs $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ y \end{pmatrix} = y \underline{\underline{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}}$

10

$$\begin{aligned} 2x + 2y &= 1 \\ x + 3y &= 3 \\ 6x + 10y &= a \end{aligned} \qquad \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 3 \\ 6 & 10 & a \end{pmatrix} \xrightarrow{l_1 \leftrightarrow l_2} \begin{pmatrix} 1 & 3 & 3 \\ 2 & 2 & 1 \\ 6 & 10 & a \end{pmatrix}$$

$$\xrightarrow{l_2 \to l_2 - 2l_1} \begin{pmatrix} 1 & 3 & 3 \\ 0 & -4 & -5 \\ 6 & 10 & a \end{pmatrix} \xrightarrow{l_3 \to l_3 - 6l_1} \begin{pmatrix} 1 & 3 & 3 \\ 0 & -4 & -5 \\ 0 & -8 & a-18 \end{pmatrix}$$

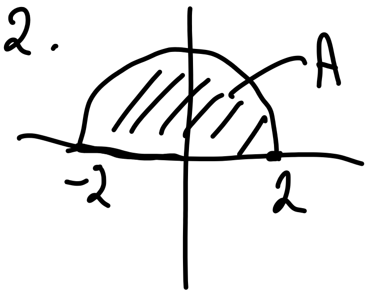
$$\xrightarrow{l_3 \to l_3 - 2l_2} \begin{pmatrix} 1 & 3 & 3 \\ 0 & -4 & -5 \\ 0 & 0 & a-18+10 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 3 \\ 0 & -4 & -5 \\ 0 & 0 & a-8 \end{pmatrix}$$

$x + 3y = 3$

$-4y = -5$ $\qquad$ dvs : $a = 8$

$$(\ 0 = a - 8\ )$$

12.




$x = r\cos\theta$

$y = r\sin\theta$

$0 \le r \le 2$

$0 \le \theta \le \pi$

$$\iint_A x^2 y^2 \, dx\, dy$$

$$= \int_0^2 \int_0^\pi r^2\cos^2\theta \, r^2 \sin^2\theta \; r \, d\theta\, dr$$

$$= \int_0^2 \int_0^\pi r^5 (\underbrace{\cos\theta \cdot \sin\theta}_{\frac{1}{2}\sin 2\theta})^2 \, d\theta\, dr$$

$$= \int_0^2 \int_0^\pi r^5 \frac{1}{4} \sin^2 2\theta \, d\theta\, dr$$

$$= \int_0^2 \int_0^\pi \frac{1}{4} r^5 \, \frac{1-\cos(4\theta)}{2} \, d\theta\, dr$$

$$= \frac{1}{8} \int_0^2 \int_0^\pi r^5 \left(1 - \cos(4\theta)\right) d\theta\, dr$$

$$= \frac{1}{8} \int_0^2 r^5 \left[\theta - \frac{1}{4}\sin(4\theta)\right]_0^\pi dr$$

$$= \frac{1}{8} \int_0^2 r^5 (\pi - 0 - 0 + 0)\, dr$$

$$= \frac{\pi}{8} \int_0^2 r^5 \, dr$$

$$= \frac{\pi}{8} \left[\frac{1}{6} r^6\right]_0^2 = \frac{\pi}{8} \frac{1}{6} 2^6$$

$$= \underline{\underline{\frac{4\pi}{3}}}$$

$$\cos(2(2\theta))$$

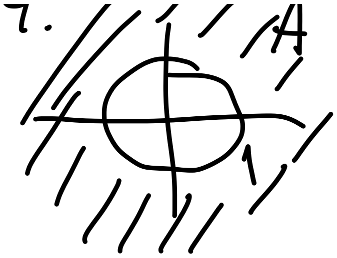
$$= \cos^2(2\theta) - \sin^2(2\theta)$$

$$= 1 - \sin^2(2\theta) - \sin^2(2\theta)$$

$$= 1 - 2\cdot\sin^2(2\theta)$$

$$\Rightarrow \sin^2(2\theta) = \frac{1-\cos(4\theta)}{2}$$

17.




$$\iint_A \frac{1}{(x^2+y^2)^3}\,dx\,dy$$

$x = r\cos\theta$
$y = r\sin\theta$

$1 \le r \le b \to \infty$
$0 \le \theta \le 2\pi$
$x^2+y^2 = r^2$

$$\int_0^{2\pi}\int_1^b \frac{1}{(r^2)^3}\, r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_1^b r^{-5}\,dr\,d\theta$$

$$= \int_0^{2\pi}\left[\frac{1}{-4} r^{-4}\right]_1^b d\theta$$

$$= \int_0^{2\pi} -\frac{1}{4}(b^{-4}-1)\,d\theta$$

$$= -\frac{2\pi}{4}(\underbrace{b^{-4}}_{\to 0}-1) \xrightarrow[b\to\infty]{} \underline{\underline{\frac{\pi}{2}}}$$

## 2018

1. $\bar{r}(t) = (t^3, e^{-2t}) \quad \bar{a}(1) = ?$

$\bar{v}(t) = \bar{r}'(t) = (3t^2, -2e^{-2t})$

$\bar{a}(t) = \bar{r}''(t) = (6t, 4e^{-2t}) \quad \bar{a}(1) = \underline{\underline{(6, 4e^{-2})}}$ $\boxed{E}$

2. $\bar{F}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x^2+3\\xy+2\end{pmatrix}$ Lineariseruing i $\begin{pmatrix}1\\1\end{pmatrix}$

$$\bar{F}'\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}2x & 0\\ y & x\end{pmatrix}$$

$$\bar{F}\begin{pmatrix}1\\1\end{pmatrix} + \bar{F}'\begin{pmatrix}1\\1\end{pmatrix}\cdot\begin{pmatrix}x-1\\y-1\end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x-1 \\ y-1 \end{pmatrix} =$$

$$= \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} 2x-2 \\ x-1+y-1 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} 2x \\ x+y \end{pmatrix} + \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

$$\boxed{= \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}}$$

$\boxed{B}$

3. $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$ 1. og 3. koordinat er like i alle lineærkombinasjoner

$\boxed{B}$ $\begin{pmatrix} -2 \\ 15 \\ -2 \end{pmatrix}$

4. $A = \begin{pmatrix} 2 & 5 & 1 \\ 4 & 16 & 3 \\ -2 & 7 & 1 \end{pmatrix}$

(Ax=0 har alltid løsning x=0)

- A 3 pivotsøyler
- Ax=0 har kun x=0 som løsning

$$\xrightarrow{l_3 \to l_3 + l_1} \begin{pmatrix} 2 & 5 & 1 \\ 4 & 16 & 3 \\ 0 & 12 & 2 \end{pmatrix} \xrightarrow{l_2 \to l_2 - 2l_1} \begin{pmatrix} 2 & 5 & 1 \\ 0 & 6 & 1 \\ 0 & 12 & 2 \end{pmatrix}$$

$$\xrightarrow{l_3 \to l_3 - 2l_2} \begin{pmatrix} 2 & 5 & 1 \\ 0 & 6 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

To pivotsøyler $\boxed{D}$

5. $A = \begin{pmatrix} 7 & -2 \\ 4 & 1 \end{pmatrix}$ $\chi_A(\lambda) = \det \begin{pmatrix} 7-\lambda & -2 \\ 4 & 1-\lambda \end{pmatrix}$

$$= (7-\lambda)(1-\lambda) - (-2)\cdot 4$$

$$= \lambda^2 - 8\lambda + 7 + 8 = \lambda^2 - 8\lambda + 15 = 0$$

abc-formel: $\lambda = \frac{8 \pm \sqrt{64-60}}{2} = \frac{8 \pm 2}{2} = \begin{cases} 5 \\ 3 \end{cases}$

$\boxed{D}$

6. $\bar{F}(x,y,z) = \left(yz\cos(xy),\ xz\cos(xy),\ \sin(xy)\right)$

$$\varphi$$

$$\frac{\partial}{\partial x} \qquad \frac{\partial}{\partial y} \qquad \frac{\partial}{\partial z}$$

$$yz\cdot\cos(xy)$$

$$\int yz\cos(xy)\,dx = \cancel{y}z\frac{1}{\cancel{y}}\sin(xy) + h(y,z)$$

Kandidat $\varphi(x,y,z) = z\cdot\sin(xy) + h(y,z)$

$$\frac{\partial\varphi}{\partial y} = xz\cdot\cos(xy) + \frac{\partial h}{\partial y} = xz\cdot\cos(xy)$$

$$\Rightarrow \frac{\partial h}{\partial y} = 0 \quad \text{dvs } h(z)$$

Kandidat $\varphi(x,y,z) = z\cdot\sin(xy) + h(z)$

$$\frac{\partial\varphi}{\partial z} = \sin(xy) + h'(z) = \sin(xy)$$

$$\Rightarrow h'(z) = 0 \Rightarrow h(z) = C$$

$$\varphi(x,y,z) = z\cdot\sin(xy) + \cancel{C}$$

$\boxed{C}$

7. $C: \bar{r}(t) = (t, t^2, t^3) \quad 0 \le t \le 1$

$$\bar{F}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad \bar{F}(\bar{r}(t)) = \begin{pmatrix} t \\ t^2 \\ t^3 \end{pmatrix}$$

$$\int_C \bar{F} \cdot d\bar{r} = \int_0^1 (t, t^2, t^3) \cdot (1, 2t, 3t^2)\, dt$$

$$\boxed{\bar{r}'(t) = (1, 2t, 3t^2)}$$

$$= \int_0^1 t + 2t^3 + 3t^5 \, dt$$

$$= \left[\frac{1}{2}t^2 + \frac{2}{4}t^4 + \frac{3}{6}t^6\right]_0^1$$

$$= \frac{1}{2} + \frac{2}{4} + \frac{3}{6} = \underline{\underline{\frac{3}{2}}} \qquad \boxed{D}$$

8. $\bar{F} = (P, Q)$ definert på område $A$

A) $\dfrac{\partial Q}{\partial x} = \dfrac{\partial P}{\partial y} \Rightarrow \bar{F}$ konservativt Nei

B) $A$ enheltsammenhengende $\Rightarrow \oint_C \bar{F} \cdot d\bar{r} = 0$ Nei

c) $A$ ikke er enheltsammenhengende $\Rightarrow \bar{F}$ ikke konservativt Nei

D) $A$ enkeltsammenhengde og $\dfrac{\partial Q}{\partial x} = \dfrac{\partial P}{\partial y}$ på hele $A \Rightarrow \bar{F} = \nabla\varphi$ Ja

E) $\boxed{D}$

) Nei. $\boxed{D}$

9. $$4x^2+y^2+24x-4y+24=0$$

$$4x^2+24x+y^2-4y=-24$$

$$4(x^2+6x+9)+(y^2-4y+4)=-24+36+4$$

$$4(x+3)^2+(y-2)^2=16 \quad \Big| \frac{1}{16}$$

$$\frac{(x+3)^2}{2^2}+\frac{(y-2)^2}{4^2}=1$$

Ellipse med sentrum i $(-3,2)$, halvakser $2, 4$

$\boxed{A}$

10. $a, b > 0 \quad \lambda_1 = 1$

D) $\begin{pmatrix} \underline{1} & a \\ 0 & \underline{b} \end{pmatrix}$ $\boxed{D}$

11. $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

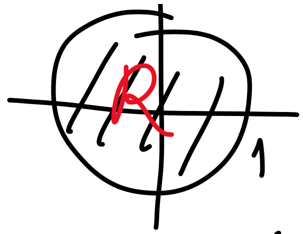
$A\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}$ Nei

$A\begin{pmatrix}1\\-1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}$ Nei

$A\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}3\\3\\3\end{pmatrix} = 3\begin{pmatrix}1\\1\\1\end{pmatrix}$ Ja. $\boxed{C}$

To siste Nei

12.




$x = r\cos\theta$

$y = r\sin\theta$

$0 \le \theta \le 2\pi$

$0 \le r \le 1$

$x^2+y^2 = r^2$

$$\iint_R x^2+y^2 \, dx\,dy = \int_0^{2\pi}\int_0^1 r^2 r \, dr\, d\theta$$

$$= 2\pi \int_0^1 r^3 dr = 2\pi \cdot \frac{1}{4} = \frac{\pi}{2}$$

$\boxed{C}$

13. $$\iint_{\mathbb{R}^2} (x^2+y^2) e^{-(x^2+y^2)^2} dx\, dy$$

Polarkoordinaten

$0 \le r \le b \to \infty$

$$\int_0^{2\pi}\int_0^b r^2 e^{-r^4} r \, dr\, d\theta$$

$$= 2\pi \int_0^b r^3 e^{-r^4} dr$$

$u = -r^4$

$du = -4r^3 dr$

$r = b \Rightarrow u = -b^4$

$r = 0 \Rightarrow u = 0$

$$= 2\pi \int_0^{-b^4} -\frac{1}{4} e^u du$$

$$= -\frac{2\pi}{4}\left(e^{-b^4} - b^0\right)$$

$$= -\frac{\pi}{2}\left(e^{-b^4} - 1\right) \xrightarrow[b\to\infty]{} \frac{\pi}{2}$$

$e^{-b^4} \to 0$

$\boxed{C}$

15. $$F(x,y) = (\overset{P}{y+\sin(x^2)}, \overset{Q}{-5x-\cos(y^2)})$$

$C: (x-3)^2+(y-3)^2 = 1$ ↺

$\frac{\partial Q}{\partial x} = -5 \quad \frac{\partial P}{\partial y} = 1$

$\partial S$"     $\frac{}{\partial x} = -5 \quad \frac{}{\partial y} = 1$

$$\oint_C \vec{F} \cdot dr = \iint_S -5 - 1 \, dx\, dy$$

$$= -6 \iint_S dx\, dy = -6 \text{ arealet}(S)$$

$$= \underline{\underline{-6\pi}}$$

$\boxed{E}$