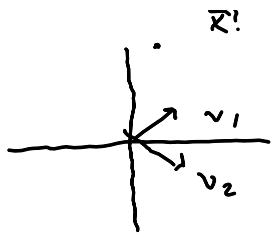


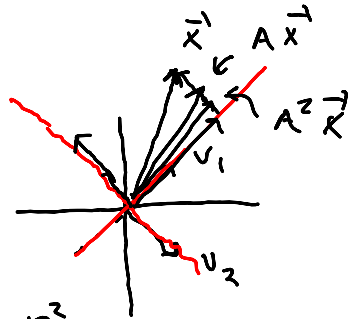
EGENVERDIER.

$$A = \begin{bmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{bmatrix}$$

$$A^n = \underbrace{A \cdot A \cdot \ldots \cdot A}_{n\text{-ganger}} \quad ?$$

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

$$A\vec{v}_1 = \vec{v}_1 . \quad A\vec{v}_2 = \frac{1}{2}\vec{v}_2 .$$



La $\vec{x} \in \mathbb{R}^2$.

Da kan vi skrive

$$\vec{x} = c_1 \cdot \vec{v}_1 + c_2 \cdot \vec{v}_2 , \quad c_1, c_2 \in \mathbb{R}^2 .$$

Vi har

$$\begin{aligned} A\vec{x} &= A \cdot (c_1 \cdot \vec{v}_1 + c_2 \cdot \vec{v}_2) \\ &= c_1 \cdot A\vec{v}_1 + c_2 \cdot A\vec{v}_2 \\ &= c_1 \cdot \vec{v}_1 + c_2 \frac{1}{2} \cdot \vec{v}_2 . \\ A^2\vec{x} &= A \left(c_1 \vec{v}_1 + c_2 \frac{1}{2} \vec{v}_2 \right) \\ &= c_1 \cdot A\vec{v}_1 + c_2 \cdot \frac{1}{2} \underline{A\vec{v}_2} \\ &= c_1 \cdot A\vec{v}_1 + c_2 \left(\frac{1}{2}\right)^2 \vec{v}_2 \end{aligned}$$

DEF 1.9.5. La $T: \mathbb{R}^n \to \mathbb{R}^n$ være en lineær avbilding. En vektor $\vec{v} \neq 0$ kalles en <u>egenvektor</u> dersom det fins $\lambda \in \mathbb{R}$ s.a. $T(\vec{v}) = \lambda \vec{v}$. I så fall kaller vi λ en egenverdi.

Merk at dersom $\vec{v}_1, \ldots, \vec{v}_k$ er egenvektorer for T med egenverdier $\lambda_1, \ldots, \lambda_k$, for en vektor $\vec{x} = c_1 \cdot \vec{v}_1 + c_2 \cdot \vec{v}_2 + \ldots + c_k \cdot \vec{v}_k$ har vi

$$T^n \vec{x} = c_1 \lambda_1^n \vec{v}_1 + c_2 \lambda_2^n \vec{v}_2 + \ldots + c_k \lambda_k^n \vec{v}_k .$$

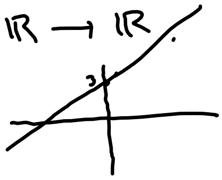
1.10 Affinavbildinger

DEF 1.10.1. En avbilding $F: \mathbb{R}^n \to \mathbb{R}^m$ sies å være en affinavbilding dersom

$$F(\vec{x}) = A\vec{x} + \vec{b}$$

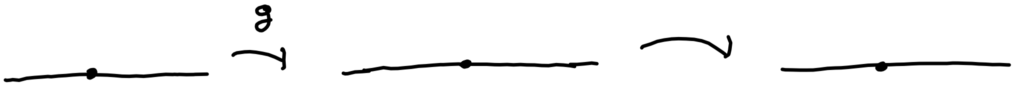
for en matrise A og $\vec{b} \in \mathbb{R}^m$.

EKS: $f(x) = 2x + 3$, $f: \mathbb{R} \to \mathbb{R}$.



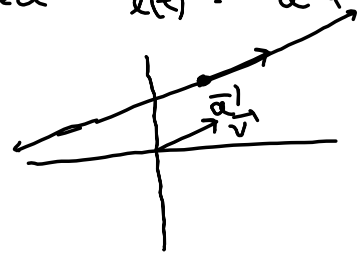
Tenk på dette som en komposisjon av $g(x) = 2x$ og en translasjon $h(x) = x + 3$

$$\xrightarrow{g} \qquad \longrightarrow$$



$\mathbb{R}$

For $\vec{a} \in \mathbb{R}^n$ og $v \in \mathbb{R}^n$ sies vi at $l: \mathbb{R} \to \mathbb{R}^n$ gitt ved $l(t) = \vec{a} + t \cdot \vec{v}$ er en parametrisert linje.



Proposisjon 1.10.2. La $F: \mathbb{R}^n \to \mathbb{R}^m$, $F = A + \vec{b}$, være en affinavbilding og la $l(t) = \vec{a} + t\vec{v}$ være en parametrisert linje. Dersom $A\vec{v} \neq 0$ så er $F(l(t))$ en parametrisert linje i $\mathbb{R}^m$ i retning $A\vec{v}$.

Bevis:

$$\begin{aligned} F(\vec{a} + t\vec{v}) &= A(\vec{a} + t\vec{v}) + \vec{b} \\ &= A\vec{a} + tA\vec{v} + \vec{b} \\ &= F(\vec{a}) + t \cdot A\vec{v}. \end{aligned}$$

$\blacksquare$

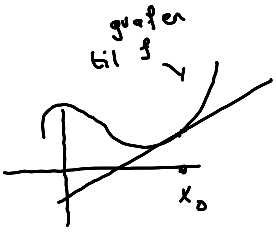
2.7/2.8 Linearisering og kjerneregelen.

Repetisjon. La $f: \mathbb{R} \to \mathbb{R}$ være en funksjon. Husk at f sies å være <u>deriverbar</u> i $x_0 \in \mathbb{R}$ dersom

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = f'(x_0)$$

eksisterer. Anta så at f er deriverbar i $x_0 \in \mathbb{R}$.
En konsekvens er at affinavbildingen

$$F(x) = f(x_0) + f'(x_0)(x - x_0)$$



er en veldig god tilnærming til f nær x_0.

Se på

$$\sigma(x) = f(x) - \overbrace{\left(f(x_0) + f'(x_0)(x - x_0)\right)}^{F(x)}$$

Vi har at

$$\lim_{x \to x_0} \frac{\sigma(x)}{x - x_0} = \lim_{x \to x_0} \left(\underbrace{\frac{f(x) - f(x_0)}{x - x_0}}_{\downarrow \, f'(x_0)} - \frac{f'(x_0)\cancel{(x - x_0)}}{\cancel{x - x_0}} \right)$$

$$= 0.$$

DEF: Dersom $B \subset \mathbb{R}^n$, $C \subset \mathbb{R}^m$, og $F: B \to C$ er en funksjon sier vi at F er <u>deriverbar</u> i $\vec{a} \in B$ dersom det fins en $(m \times n)$-matrise A s.a.

$$F(\vec{x}) = F(\vec{a}) + A \cdot (\vec{x} - \vec{a}) + \sigma(\vec{x}),$$

der $\dfrac{\sigma(\vec{x})}{\|\vec{x} - \vec{a}\|} \to 0$ når $\vec{x} \to \vec{a}$.

(Merk: $F(\vec{a}) + A(\vec{x} - \vec{a}) = F(\vec{a}) + A\vec{x} - A\vec{a}$
$= A\vec{x} + (F(\vec{a}) - A\vec{a})$.)

Vi skriver gjerne $F'(\vec{a})$ for denne matrisen.

I så fall er A Jacobi-matrisen til F, det vil si

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(\vec{a}) & \frac{\partial f_1}{\partial x_2}(\vec{a}) & \dots & \frac{\partial f_1}{\partial x_n}(\vec{a}) \\ \vdots & & & \vdots \\ \frac{\partial f_m}{\partial x_1}(\vec{a}) & \frac{\partial f_m}{\partial x_2}(\vec{a}) & \dots & \frac{\partial f_m}{\partial x_n}(\vec{a}) \end{bmatrix},$$

dersom $F = (f_1, \dots, f_m)$.

<u>DEF 2.8.2</u> Avbildingen

$$T_{\vec{a}} F(\vec{x}) = F(\vec{a}) + F'(\vec{a})(\vec{x} - \vec{a})$$

kalles <u>lineariseringen</u> til F i $\vec{a}$.

<u>Eks 1:</u> For $f : \mathbb{R}^n \to \mathbb{R}$, $\vec{a} \in \mathbb{R}^n$:

$$F'(\vec{a}) = \left[\frac{\partial f}{\partial x_1}(\vec{a}) \; .. \; \frac{\partial f}{\partial x_n}(\vec{a}) \right]$$

$$\vec{x} - \vec{a} = \begin{bmatrix} x_1 - a_1 \\ x_2 - a_2 \\ \vdots \\ x_n - a_n \end{bmatrix}$$

$$F'(\vec{a}) \cdot (\vec{x} - \vec{a}) = \frac{\partial f}{\partial x_1}(\vec{a})(x_1 - a_1) + \dots + \frac{\partial f}{\partial x_n}(\vec{a})(x_n - a_n)$$

$$= \nabla f(\vec{a}) \cdot (\vec{x} - \vec{a}).$$

Prikkprodukt:
$\vec{x} \cdot \vec{y}$
$= x_1 y_1 + x_2 y_2 + \dots + x_n y_n$.

$$\boxed{T_{\vec{a}} f(x) = f(a) + \nabla f(\vec{a}) \cdot (\vec{x} - \vec{a})}.$$