Forelesning 2. april

Oppgavene fra midtveis

Oppgave 1

$$\vec{F}(x,y) = \underbrace{\begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$A = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}, \quad \det A = 3 \cdot 4 - 1 \cdot 2 = 12 - 2 = 10$$

$$\text{areal } F(R) = \text{areal}(R) \cdot \det(A) = 2 \cdot 2 \cdot 10 = \underline{40}$$

Oppgave 2

$$F'\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x & 2y & 0 \\ 3x^2 & 0 & 2z \end{pmatrix} \qquad G'\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x & 3y^2 \\ 3x^2 & 3y^2 \\ 3x^2 & 2y \end{pmatrix}$$

$$G(1,-1) = (0,0,2)$$

$$H'(1,-1) = F'(0,0,2) \cdot G'(1,-1)$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 3 & 3 \\ 3 & -2 \end{pmatrix} = \underline{\begin{pmatrix} 0 & 0 \\ 12 & -8 \end{pmatrix}}$$

Oppgave 3

$$\vec{F}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^2 + y \\ x + y \\ x + y^2 \end{pmatrix} \qquad \vec{F}'\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x & 1 \\ 1 & 1 \\ 1 & 2y \end{pmatrix}$$

$$\vec{F}'\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ 1 & 2 \end{pmatrix} \qquad \vec{F}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

$$T_{\vec{a}}\vec{F} = \vec{F}(\vec{a}) + \vec{F}'(\vec{a})(\vec{x} - \vec{a})$$

$$= \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}\left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right)$$

$$= \begin{pmatrix} 2x + y + 2 - 2 - 1 \\ x + y + 2 - 1 - 1 \\ x + 2y + 2 - 1 - 2 \end{pmatrix} = \underline{\begin{pmatrix} 2x + y - 1 \\ x + y \\ x + 2y - 1 \end{pmatrix}}$$

Oppgave 4

$$\vec{r}(t) = (\sin t, \sqrt{2}\cos t, \sin t)$$
$$\vec{r}\,'(t) = (\cos t, -\sqrt{2}\sin t, \cos t)$$
$$v(t) = \sqrt{\cos^2 t + 2\sin^2 t + \cos^2 t}$$
$$= \sqrt{2(\cos^2 t + \sin^2 t)} = \sqrt{2}$$

buelengde: $\displaystyle\int_0^{\frac{\pi}{2}} v(t)\,dt = \frac{\pi}{2}\cdot\sqrt{2} = \pi/\sqrt{2}$

Oppgave 5

$$\vec{r}(t) = (t, t^2, t^3) \Rightarrow v(t) = \sqrt{1^2 + (2t)^2 + (3t^2)^2}$$
$$= \sqrt{1 + 4t^2 + 9t^4}$$
$$a(t) = v'(t) = \frac{8t + 36t^3}{2\sqrt{1+4t^2+9t^4}} = \frac{4t + 18t^3}{\sqrt{1+4t^2+9t^4}}$$

Oppgave 6

$$\vec{r}(t) = (e^t, 2\sqrt{2}e^t, 4e^t)$$
$$\vec{r}\,'(t) = (e^t, 2\sqrt{2}e^t, 4e^t) \quad v(t) = e^t|(1, 2\sqrt{2}, 4)|$$
$$= e^t\sqrt{1+8+16}$$
$$= 5e^t$$

$$f(x,y,z) = \sqrt{x^2+y^2+z^2} = |\vec{r}(t)|$$
$$= |\vec{r}\,'(t)| = 5e^t$$

$$\int_C f\,ds = \int_0^{\ln 2} 5e^t\, 5e^t\,dt = \int_0^{\ln 2} 25e^{2t}\,dt$$
$$= \left[\frac{25}{2}e^{2t}\right]_0^{\ln 2}$$
$$= \frac{25}{2}e^{2\ln 2} - \frac{25}{2}$$
$$= \frac{25}{2}e^{\ln 4} - \frac{25}{2}$$
$$= \frac{25}{2}\cdot 4 - \frac{25}{2} = \frac{75}{2}$$

Oppgave 7

$$\vec{r}(t) = (-\cos t, \sin t, t^2)$$
$$\vec{r}\,'(t) = (\sin t, \cos t, 2t)$$
$$\vec{F}(\vec{r}(t)) = (\sin t, \cos t, 2t^2 - \cos t + \sin t)$$

$$\vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t) = \sin^2 t + \cos^2 t + 4t^3 - 2t\cos t + 2t\sin t$$

$$= 1 + 4t^3 - 2t\cos t + 2t\sin t$$

$$\int_0^{2\pi} t\cos t\,dt = \left[t\sin t\right]_0^{2\pi} - \int_0^{2\pi}\sin t\,dt = 0$$

$$\int_0^{2\pi} t\sin t\,dt = \left[-t\cos t\right]_0^{2\pi} + \int_0^{2\pi}\cos t\,dt = -2\pi$$

$$\int_0^{2\pi}\left(1 + 4t^3 \cancel{- 2t\cos t} + 2t\sin t\right)dt$$

$$= 2\pi + \left[t^4\right]_0^{2\pi} + 2(-2\pi)$$

$$= 16\pi^4 - 2\pi$$

Oppgave 8

$$\frac{\partial\phi}{\partial x} = 3x^2 + y + z \Rightarrow \phi(x,y,z) = x^3 + xy + xz + C_1(y,z)$$

$$\frac{\partial\phi}{\partial y} = 3y^2 + x + z \Rightarrow \phi(x,y,z) = y^3 + xy + yz + C_2(x,z)$$

$$\frac{\partial\phi}{\partial z} = 3z^2 + x + y \Rightarrow \phi(x,y,z) = z^3 + xz + yz + C_3(x,y)$$

ser at $\phi(x,y,z) = x^3 + y^3 + z^3 + xy + xz + yz + \text{konst}$ er potensial funksjon.

$$C_1(y,z) = y^3 + z^3 + yz + \text{konst}$$

$$C_2(x,z) = x^3 + z^3 + xz + \text{konst}$$

$$C_3(x,y) = x^3 + y^3 + xy + \text{konst.}$$

Oppgave 9

$$4x^2 + 4x + y^2 + 6y + 6 = 0$$

$$4\left(x^2 + x + \tfrac{1}{4}\right) + \left(y^2 + 6y + 9\right) = -6 + 1 + 9$$

$$4\left(x + \tfrac{1}{2}\right)^2 + (y+3)^2 = 4$$

$$\left(x + \tfrac{1}{2}\right)^2 + \frac{(y+3)^2}{2^2} = 1$$

ellipse med sentrum $\left(-\tfrac{1}{2}, -3\right)$, halvakser 2 og 1

Oppgave 10

$$\begin{pmatrix} 1 & 3 & 1 \\ 2 & 4 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 1 \\ 0 & -2 & -1 \\ 0 & -1 & -1 \\ 0 & -3 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & -3 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Vi ser at siste søyle er en pivotsøyle, slik at $\begin{pmatrix} 1 \\ 1 \\ 0 \\ 2 \end{pmatrix}$ ikke kan skrives som linear. komb. av de to andre.

Oppgave 11

$$A = \begin{pmatrix} 2 & 6 & 1 \\ 4 & 12 & 4 \\ 1 & 3 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 2 \\ 0 & 0 & -4 \\ 0 & 0 & -3 \end{pmatrix} \Rightarrow \text{1. og 3. søyle er pivotsøyler.}$$

$$\sim \begin{pmatrix} 1 & 3 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Oppgave 12

$$\begin{vmatrix} 2 & 3 & -2 & 1 \\ 0 & 1 & 3 & 0 \\ 1 & -1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix} = -(1) \begin{vmatrix} 0 & 1 & 3 \\ 1 & -1 & 2 \\ 0 & 0 & 1 \end{vmatrix} = -(1)\cdot 1 \cdot \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix}$$

$$= -\left(0\cdot(-1) - 1\cdot 1\right)$$

$$= -(-1) = \underline{1}$$

Oppgave 13

$$\det(\lambda I - A) = \begin{vmatrix} \lambda-1 & -1 & -2 \\ 0 & \lambda-2 & 0 \\ -2 & -1 & \lambda-1 \end{vmatrix} = (\lambda-2)\begin{vmatrix} \lambda-1 & -2 \\ -2 & \lambda-1 \end{vmatrix}$$

$$= (\lambda-2)\left((\lambda-1)^2 - 4\right)$$

$$= (\lambda-2)(\lambda-3)(\lambda+1).$$

$(\lambda-1)^2 = 4$

$\lambda - 1 = \pm 2$

$\lambda = 3$ eller $\lambda = -1$

egenverdier: 2, 3, −1

Oppgave 14

$$\det(\lambda I - A) = \begin{vmatrix} \lambda - 0.8 & -0.7 \\ -0.2 & \lambda - 0.3 \end{vmatrix} = (\lambda-0.8)(\lambda-0.3) - 0.14$$
$$= \lambda^2 - 1.1\lambda + 0.1$$

dette er 0 når $\lambda = \dfrac{1.1 \pm \sqrt{1.21 - 0.4}}{2} = \dfrac{1.1 \pm 0.9}{2} \Rightarrow \lambda_1 = 0.1,\ \lambda_2 = 1$

$\lambda = 0.1$: $\lambda I - A = \begin{pmatrix} -0.7 & -0.7 \\ -0.2 & -0.2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \Rightarrow x_1 + x_2 = 0$
$\Rightarrow x_2 = -x_1$

egenvektor: $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\lambda = 1$: $\lambda I - A = \begin{pmatrix} 0.2 & -0.7 \\ -0.2 & 0.7 \end{pmatrix} \sim \begin{pmatrix} 2 & -7 \\ 0 & 0 \end{pmatrix} \Rightarrow 2x_1 - 7x_2 = 0$

sett f.eks: $x_1 = 7,\ x_2 = 2$

egenvektor: $\begin{pmatrix} 7 \\ 2 \end{pmatrix}$

Oppgave 15

$$\begin{pmatrix} 1 & 3 & 2 & 2 \\ 4 & 2 & 1 & 4 \\ 3 & -1 & -1 & a \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 2 & 2 \\ 0 & -10 & -7 & -4 \\ 0 & -10 & -7 & a-6 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 2 & 2 \\ 0 & -10 & -7 & -4 \\ 0 & 0 & 0 & a-2 \end{pmatrix}$$

Vi ser at siste søyle er pivotsøyle $\Leftrightarrow$ $a \neq 2$

Derfor: systemet har kun løsning for $a = 2$.