Kap. 14 : Komplekse vektorrom (Lay 5.5)

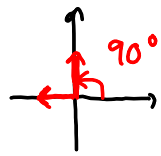
Viste KOLA kap 14 : Kun 14.1 pensum

eks. Finne egenvektorar og egenverdier for

$$M = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$M\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$M\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$



Egenverdier

$$\begin{vmatrix} 0-\lambda & -1 \\ 1 & 0-\lambda \end{vmatrix} = \lambda^2 + 1 = 0 \quad \text{gir}$$

$$\lambda = \frac{-0 \pm \sqrt{0-4}}{2} = \frac{-0 \pm \sqrt{-1}\cdot\sqrt{4}}{2} = \pm \frac{i \cdot 2}{2} = \begin{cases} i \\ -i \end{cases}$$

Egenvektorer til $\lambda_1 = i$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = i \begin{bmatrix} a \\ b \end{bmatrix} \quad \text{gir} \quad \begin{cases} -b = ia \\ a = ib \end{cases} \quad \left(\begin{matrix} \text{Begge sier} \\ a = ib \end{matrix}\right)$$

$$\text{Basis} \quad \begin{bmatrix} i \\ 1 \end{bmatrix}$$

Egenvektorer til $\lambda_2 = -i$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = -i \begin{bmatrix} a \\ b \end{bmatrix} \quad \text{gir} \quad \begin{cases} -b = -ia \\ a = -ib \end{cases} \quad \left(\begin{matrix} \text{Begge sier} \\ a = -ib \end{matrix}\right)$$

$$\text{Basis} \quad \begin{bmatrix} -i \\ 1 \end{bmatrix}$$

Oppgave 14.1.7 (KOLA) (Setter opp til neste uke!)

$$\begin{cases} x_{n+1} = 50x_n - y_n \\ y_{n+1} = 50y_n + x_n \end{cases}$$

x_n : Slemme fluer

y_n : Snille —"—

a) $\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} 50 & -1 \\ 1 & 50 \end{bmatrix} \cdot \begin{bmatrix} x_n \\ y_n \end{bmatrix}$ for $n \geq 0$, dvs. $A = \begin{bmatrix} 50 & -1 \\ 1 & 50 \end{bmatrix}$

b) Egenverdier for A:

$$\begin{vmatrix} 50-\lambda & -1 \\ 1 & 50-\lambda \end{vmatrix} = (\lambda - 50)^2 + 1 = \lambda^2 - 100\lambda + 2500 + 1$$
$$= \lambda^2 - 100\lambda + 2501 = 0$$

gir $\lambda = \dfrac{100 \pm \sqrt{100^2 - 4 \cdot 2501}}{2} = \dfrac{100 \pm \sqrt{-4}}{2}$

$$= \frac{100 \pm \sqrt{-1} \cdot \sqrt{4}}{2} = 50 \pm i$$

Egenvektorer til $\lambda_1 = 50 + i$

$\begin{bmatrix} 50 & -1 \\ 1 & 50 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = (50+i) \begin{bmatrix} a \\ b \end{bmatrix}$ gir $\begin{cases} \cancel{50}a - b = (\cancel{50} + i)a \\ a + \cancel{50}b = (\cancel{50} + i)b \end{cases}$

(Begge sier $b = -ia$) Løsn: $\begin{bmatrix} A_1 \\ -iA_1 \end{bmatrix} = A_1 \cdot \underbrace{\begin{bmatrix} 1 \\ -i \end{bmatrix}}_{\text{basis}}$

Egenvektorer til $\lambda_2 = 50 - i$

$\begin{bmatrix} 50 & -1 \\ 1 & 50 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = (50-i) \begin{bmatrix} a \\ b \end{bmatrix}$ gir $\begin{cases} \cancel{50}a - b = \cancel{50}a - ia \\ a + \cancel{50}b = \cancel{50}b - ib \end{cases}$

(Begge sier $b = ia$) Løsn: $\begin{bmatrix} A_2 \\ iA_2 \end{bmatrix} = A_2 \cdot \begin{bmatrix} 1 \\ i \end{bmatrix}$

c) $\begin{bmatrix}100\\2\end{bmatrix} = \begin{bmatrix}x_0\\y_0\end{bmatrix} = \begin{bmatrix}A_1\\-iA_1\end{bmatrix} + \begin{bmatrix}A_2\\iA_2\end{bmatrix}$ gir $\begin{cases}100 = A_1 + A_2 & \text{I}\\ 2 = -iA_1 + iA_2 & \text{II}\end{cases}$

$\text{I}: \quad A_1 = 100 - A_2$

$\text{II}: \quad 2 = -i(100 - A_2) + iA_2 = -100i + 2iA_2$

dvs. $\quad 2 + 100i = 2iA_2$

$$A_2 = \frac{2+100i}{2i} = \frac{1}{i} + 50 = \frac{-i}{-i^2} + 50$$

$$A_2 = 50 - i$$

I gir da: $A_1 = 100 - (50 - i) = 50 + i$

Konklusjon:

$$\begin{bmatrix}100\\2\end{bmatrix} = \begin{bmatrix}50+i\\-i(50+i)\end{bmatrix} + \begin{bmatrix}50-i\\50i-i^2\end{bmatrix} = \begin{bmatrix}50+i\\1-50i\end{bmatrix} + \begin{bmatrix}50-i\\1+50i\end{bmatrix}$$

(ok!)

Så

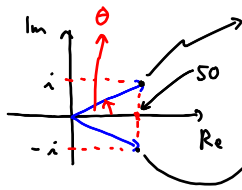
$$\begin{bmatrix}x_n\\y_n\end{bmatrix} = A^n \cdot \begin{bmatrix}x_0\\y_0\end{bmatrix} = A^n \begin{bmatrix}100\\2\end{bmatrix} = A^n\left(\begin{bmatrix}50+i\\1-50i\end{bmatrix} + \begin{bmatrix}50-i\\1+50i\end{bmatrix}\right)$$

$$= A^n\begin{bmatrix}50+i\\1-50i\end{bmatrix} + A^n\begin{bmatrix}50-i\\1+50i\end{bmatrix}$$

$$= (50+i)^n\begin{bmatrix}50+i\\1-50i\end{bmatrix} + (50-i)^n\begin{bmatrix}50-i\\1+50i\end{bmatrix}$$

Altså: $\quad \underline{x_n = (50+i)^{n+1} + (50-i)^{n+1}} \quad$ for $n \geq 0$

d)



$50 + i = \sqrt{50^2+1^2}\, e^{i\theta}$

$50 - i = \sqrt{50^2+1^2}\, e^{i(-\theta)}$

der $\tan\theta = \frac{1}{50}$ dvs. $\theta = \arctan\frac{1}{50} \approx 0{,}02$ (rad)

Så $\quad x_n = \left(\sqrt{2501}\, e^{i\theta}\right)^{n+1} + \left(\sqrt{2501}\, e^{-i\theta}\right)^{n+1}$

$$= \left(\sqrt{2501}\right)^{n+1} \cdot \left[\left(e^{i\theta}\right)^{n+1} + \left(e^{-i\theta}\right)^{n+1}\right]$$

$$= \left(\sqrt{2501}\right)^{n+1} \cdot \left[e^{i(n+1)\theta} + e^{-i(n+1)\theta}\right]$$

$$= \underline{\left(\sqrt{2501}\right)^{n+1} \cdot 2\cos\left[(n+1)\theta\right]}$$

$$\boxed{\begin{aligned} &e^{iu} + e^{-iu} \\ &= \cos u + i\cancel{\sin u} \\ &+ \cos(-u) + i\cancel{\sin(-u)} \\ &= 2\cos u \end{aligned}}$$

e) Slemme fluer utryddes når $\cos\left[(n+1)\theta\right] = 0$, dvs.
når $(n+1)\theta = \frac{\pi}{2}$, dvs. $n\theta + \theta = \frac{\pi}{2}$, $n = \dfrac{\frac{\pi}{2} - \theta}{\theta} \approx 78$

<u>De slemme fluene utryddes i følge modellen etter ca. 78 uker.</u>