

Addition formulas

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Abstract

Tentative topic for a project in MAT2000 spring 2021. Read the description and please contact me at karoline.moe@ub.uio.no if you are interested in working on one of this topic, or something tangential.

Differentiation is easy, integration is hard. In the 18th century, following the first developments of mathematical analysis, computing the arc length of quite simple algebraic curves using integrals turned out to be surprisingly difficult. This challenge led the contemporary great mathematicians, amongst others, Euler and Gauss, to develop addition formulas for integrals.

In 1826, the young Norwegian mathematician Niels Henrik Abel wrote a thesis with extremely powerful results and a general addition formula. It turned out that this famous theorem has wide-reaching consequences far beyond Abel's original intention.

The main aim of this project is to dive into Abel's addition theorem and understand some of its ingredients. A starting point would be to set up the relatively simple formulas for the arc length of an ellipse and the lemniscate and explain the implications of these, see [Aub79; Dag10; Ras92]. Moreover, a study of cubic curves in light of this theorem would be desirable [Bix98]. The further research direction of the project depends on the candidate's interest.

References

- [Aub79] Aubert, K. E. "Abel's addition theorem". In: *Normat*, no. 4 (1979), pp. 149–158, 171.
- [Bix98] Bix, R. *Conics and cubics: A concrete introduction to algebraic curves*. Undergraduate Texts in Mathematics. Springer-Verlag, New York, 1998, pp. x+289.
- [Dag10] Dagsvik, J. K. "Elliptic integrals I. An elementary approach to Abel's addition theorem". In: *Normat* vol. 58, no. 1 (2010), pp. 6–34, 48.
- [Ras92] Rasmussen, R. *Abels avhandling: "Mémoire sur une propriété générale d'une classe très-étendue de fonctions transcendentes"*. nob. Kristiansand, 1992.