Families of rational unicuspidal curves

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Abstract

Tentative topic for a project in MAT2000 spring 2021. Read the description and please contact me at karoline.moe@ub.uio.no if you are interested in working on one of this topic, or something tangential.

A rational algebraic curve in the projective plane is called unicuspidal if it has one cusp singularity. The canonical examples of such curves are given by the zero sets in $\mathbb{P}^2$ of the homogeneous polynomial

$$F = y^n + x^{n-1}z,$$

for any integer $n \geq 3$. By adding certain monomials to the polynomial F, we sometimes get families of curves with the same type of singularity (topologically), but other properties of the singularity and the curve may change in the process.

The main aim of this project is to investigate and describe some families of plane rational unicuspidal curves in terms of special points and invariants, starting out with [DS20; Mau17; MM19; Moe08]. In particular, it is desirable to find and work with defining polynomials and parametrizations for some examples of low degree and study these with [Maple].

References

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