JACOBIAN CONJECTURE AND PROPERNESS OF POLYNOMIAL MAPS

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Polynomial equations appear quite often in algebra and algebraic geometry. People surely see it at least once in their high school mathematics (quadratic and cubic equations, in one variable), and it is hidden from the side but it has been also used in cryptography (elliptic curves, and here, Niels Henrik Abel played an important role, as he did in the theory of equations of degree 5 or higher). Many questions in mathematics can be reduced to (countably many) polynomial equation systems: from questions in elementary Euclidean geometry which you see in high school (circles, lines, triangles) to long standing open questions such as: the Jacobian conjecture, whether two given algebraic varieties are birational to each other... (One long-standing 1-million-dollar open question, the Hodge conjecture, unfortunately is not yet known to be reducible to countably many polynomial equations. It is very good if you can figure out the answer.)

Jacobian conjecture asks if a polynomial map $f : \mathbb{C}^n \to \mathbb{C}^n$, whose Jacobian is every invertible, has a polynomial inverse. It is easy to state but it is very difficult. It is a long standing open question and has relations to many fields in mathematics and physics. Many wrong proofs have been published on it.

I can offer 2 directions, both related to the Jacobian conjecture:

Direction 1: Groebner Basis. In this direction, the student will use computer algebra to check whether a polynomial map is an automorphism.

Direction 2: Properness of maps. In this direction, the student will study properness of polynomial maps and applications to Jacobian conjecture.

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References: (More will be given later)

Cox, David A.; Little, J.; O'Shea, D. (1997). Ideals, Varieties, and Algorithms: An Introduction to Computational Algebraic Geometry and Commutative Algebra. Springer. ISBN 0-387-94680-2.

Tuyen Trung Truong, Some new theoretical and computational results around the Jacobian conjecture, developed and revised from preprint arXiv: 1503.08733. Accepted in International Journal of Mathematics.

Tuyen Trung Truong, Some observations on the properness of Identity plus linear powers, arXiv:2004.03309, arXiv:2005.02260.

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