

Chromatic polynomials in combinatorics and geometry

Bachelor project proposal

The chromatic polynomial of a graph is a function that counts the number of proper colourings of its vertices. This polynomial was introduced by Birkoff as an approach to prove the 4-colour theorem. The 4-colour theorem is essentially equivalent to 4 not being a root of the chromatic polynomial for a planar graph.

Though a beautiful and elegant tool, studying this polynomial has yet to lead to a proof the 4-colour theorem; the only proof of which is via a computer. However, very recently there have been many breakthroughs in the understanding of the coefficients and most of these have come from connections of graph theory to linear algebra and algebraic geometry.

This bachelors project will look at the various different ways of calculating the chromatic polynomial, including deletion and contraction methods and the Möbius inversion formula of Whitney. It will also study the recent developments in understanding the properties of the coefficients and how these techniques rely on relations to other fields of mathematics and generalise beyond graphs to matroids. Applications to solving problems in networks can also be considered.