

Special configurations of points, lines and conics

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It has been known since the 18th century that a special configuration of 9 points and 12 lines, each point lying on 4 lines and each line containing 3 points, can be realized by considering inflection points of a nonsingular projective plane curve of degree 3. This is known as the $(12_3, 9_4)$ -configuration. Moreover, the 9 points can be taken as common inflection points of a pencil of curves, the *Hesse pencil*, given by the equation

$$\lambda(x^3 + y^3 + z^3) + \mu xyz = 0.$$

A first goal of this project is to study the algebraic curves in the Hesse pencil following [AD09].

It turns out that there are other configurations of points and lines – and conics – that originate from pencils of curves with properties that resemble those of the Hesse pencil. There actually exists a series of such *Halphen pencils*, and a second goal of this project is to study the *Chilean pencil* and the $(12_6, 9_8)$ -configuration of points and conics, first described in [DLPU20].

Depending on interests and time, a third goal could be to study other configurations, in particular the dual Hessian configuration and the Hesse–Salmon configuration [Dol04].

References

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