THE RIEMANN HYPOTHESIS

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Watch the video https://www.youtube.com/watch?v=zlm1aajH6gY by Quanta Magazine (and don't be insulted by the mathematical level of the first six minutes). The purpose of this project is to understand the connection between the distribution of the prime numbers and the zeros of the Riemann zeta function

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}.$$

Here $s = \sigma + it$ is a complex number. The series is absolutely convergent if and only if $\sigma > 1$. However, it is possible to analytically extend the Riemann zeta function to all complex numbers $s \neq 1$. We say that a complex number s is a zero of ζ if

$$\zeta(s) = 0.$$

The trivial zeros of ζ are $s = -2, -4, -6, \ldots$. There are also infinitely many zeros in the critical strip $0 < \sigma < 1$. The Riemann hypothesis states that the zeros in the critical strip all lie on the critical line $\sigma = 1/2$.

Recommended previous knowledge. MAT2100/MAT2400 and MAT2410.

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