MAT2000 (Research projects in mathematics)

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I can offer 3 projects:

Project 1: Fermat's Last theorem over different rings

Fermat's last theorem (proved by Wiles) asserts that there are no non-trivial solutions to the Fermat equation

$x^n + y^n = z^n$, n>2

where x,y,z are integers. Note that the equation $x^n+y^n=z^n$ makes sense over any ring R.
The project will be to investigate what happens when R is (i) a finite field (ii) the p-adic integers and (iii) the ring of polynomials over a field R=k[t].

Project 2: The Dehn invariant

Hilbert's third problem asks whether any two polyhedra with equal volume can be dissected into each other. This was answered in the negative by Max Dehn, who showed that it was impossible to cut a cube into a finite number of polyhedral pieces that can be reassembled into a tetrahedron. The proof involved what is now called the "Dehn invariant". The project will be to survey the circle of ideas related to Hilbert's 3rd problem, and to understand Dehn's proof.

Project 3: A project in algebraic geometry (details to be decided later)