PROPERTIES OF USELESS QUANTUM STATES

Disclaimer: Although this project is elementary, it contains an open-ended research question, where the advisor himself does not know the outcome.

Large parts of quantum information theory study the properties of *quantum states*, i.e., positive semidefinite matrices of unit trace. A quantum state ρ_{AB} on $\mathbb{C}^{d_A} \otimes \mathbb{C}^{d_A}$ is called *separable* if it can be written as a convex combination $\rho_{AB} = \sum_i p_i \sigma_i^A \otimes \tau_i^B$ with quantum states σ_i^A and τ_i^B acting on $\mathbb{C}^{d_A}$ and $\mathbb{C}^{d_B}$, respectively. Quantum states that are not separable are called *entangled.* The transpose map ϑ acting on matrices by $\vartheta(X) = X^T$ (with respect to any fixed basis) gives a simple necessary criterion for quantum states to be separable: If the quantum state ρ_{AB} is separable, then $(\mathrm{id}_A \otimes \vartheta_B)(\rho_{AB})$ is positive semidefinite. We say that a quantum state has *positive partial transposition* (PPT) if this condition is satisfied. Unfortunately, not all PPT quantum states are separable and the structure of entangled PPT states turns out to be quite complicated whenever $(d_A, d_B) \notin \{(2,2),(2,3),(3,2)\}$.

Entangled PPT states have the counterintuitive property that they can be created by two parties acting locally on a pure state entangled state shared between them and communicating classically, but this entanglement cannot be recovered when restricting to the same operations. In particular, PPT states are useless for quantum communication, but it turns out that some of them are useful for other information processing tasks such as quantum cryptography.

The project aims at figuring out the entanglement theoretic and operational properties of some new classes of PPT states based on combinatorial structures (special kinds of latin squares). The objectives of the project are as follows:

(1) Learn the basics of quantum mechanics and entanglement theory to understand the problem.
(2) Study a new class of PPT states and understand how properties of the combinatorial structure imply that they are entangled.
(3) Study how entanglement measures can be evaluated on these states and whether their cryptographic potential can be determined. This could involve doing some numerics using semidefinite optimization.
(4) Optional: Further explore the properties of this or other constructions of entangled PPT states.

Prerequesites for the project are a thorough understanding of linear algebra and calculus. Knowledge in discrete mathematics and combinatorics might be helpful.

Literature:

- Starting point could be Chapter 2 in the book "Alice and Bob meet Banach" by Guillaume Aubrun and Stanislaw Szarek, which you can find online for free.
- Another starting point is the review article "Entanglement measures" by Martin Plenio and Shashank Virmani (see https://arxiv.org/pdf/quant-ph/0504163.pdf).