9-27-72

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Project description

An elliptic curve is a smooth plane curve given by the zero set of a homogeneous cubic polynomial in three variables. Such curves have tremendously many important applications in real life, for instance in cryptography.

In this project, the aim is to study elliptic curves in different forms, in particular the Hesse pencil given by

$$\lambda(x^3 + y^3 + z^3) + \mu xyz,$$

where λ, μ are the parameters of the pencil.

If a curve has a smooth point and contact order three with a line at that point, then the point is referred to as an inflection point. The clue is that the tangent line has more contact than expected with the curve at the point. It is well known that every curve in the Hesse pencil has 9 inflection points and that they have quite special properties [AD09]. A first goal in this project is to find and describe these points and their properties.

Similarly, if a curve at a smooth point has contact order six with a conic, the point is referred to as a sextactic point. The clue is that the osculating conic has more contact than expected with the curve at the point. A curve in the Hesse pencil has 27 such points. A second goal in this project is to determine and describe these points [MM19].

By Bezout's theorem on intersection between curves it is not possible to find smooth points on a cubic curve with contact order ten, i.e., higher than expected, with another cubic. However, on a curve in the Hesse pencil there are 72 points with contact order *nine* with another cubic, actually a pencil of cubics. The main goal in this project is, for a curve in the Hesse pencil, to determine these 72 points and study them and the osculating cubics more closely following [Har75] and perhaps understand how the Cayley–Bacharach Theorem comes into play. See also [BB14] and [SS22].

References

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