MAT2000: Coends in Category Theory

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Motivation: Category theory emerged in the middle of the 20th as a unifying language across multiple mathematical disciplines. Developments in the theory have allowed to describe mathematical constructions abstractly which has contributed to the translation of ideas and techniques of proof (such as duality arguments or diagram chasing) from one area of mathematics to another.

A *(co)end* is a type of categorical (co)limit which resembles *integration* for suitable functors of the form $F\colon \mathcal{C}^{\mathrm{op}} \times \mathcal{C} \longrightarrow \mathcal{D}$. This notion describes a large variety of mathematical concepts: for example group actions are an instance of such functors and their (co)ends the corresponding (co)invariants, or the tensor product of bimodules can be described as a coend, as well.

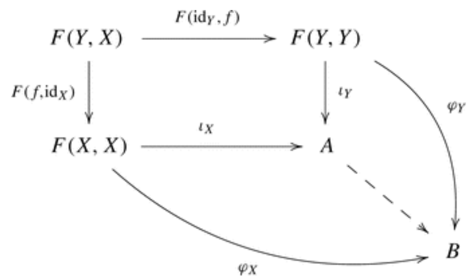

Figure 1: Universal property of a coend

Goal: The purpose of the project is to get familiarized with basic concepts in category theory and learn the notion of coends, their properties and some sample applications. In a second stage the idea is to deepen the understanding of coends by further studying one or multiple of the following topics: the reformulation of **Stokes's theorem** in terms of coends, balanced tensor product of bimodules, profunctor composition, ninja Yoneda Lemma, **Tannaka-Duality**, or a **Peter-Weyl theorem** for finite dimensional algebras, among others.

References

[Lor21] F. Loregian, *(Co)end Calculus* (Cambridge University Press, Cambridge 2021)

[McL71] S. Mac Lane, *Categories for the Working Mathematician* (Springer Verlag, New York 1971)

[Rie16] E. Riehl, *Category Theory in Context* (Dover Publications, New York 2016)