MAT2000 Project:
The Fourier transform on $L^p(\mathbb{R}^d)$

Supervised by Ola Mæhlen

The Fourier transform $\mathcal{F}\colon f \mapsto \hat{f}$ decomposes a function $f\colon \mathbb{R}^d \to \mathbb{R}$ into its pure frequencies: The map $\hat{f}\colon \mathbb{R}^d \to \mathbb{R}$ tells us how present a particular frequency $\xi \in \mathbb{R}^d$ is in f and is formally defined by

$$\hat{f}(\xi) = \int_{\mathbb{R}^d} f(x) e^{-i\xi\cdot x}\,dx.$$

This powerful transform has applications throughout mathematics and dates back to the early 19th century when the French physicist Joseph Fourier worked on developing an analytic theory for the flow of heat. This project is devoted to one of the earliest topics of study for the Fourier transform, namely its properties on L^p spaces. The theory is a cornerstone of harmonic analysis and has application in PDE theory.

The project will first explore the properties of $\mathcal{F}$ on L^1 functions and develop the basics theory including the Riemann–Lebesgue lemma and convolution theorems. Next is the theory for L^2 functions: The Fourier inversion theorem and the Plancherel theorem will be proved. Then, using the Marcinkiewicz interpolation theorem, the Fourier transform will be extended to L^p spaces in between L^1 and L^2 and, if time permits, some results on L^p spaces for $p > 2$ will be explored as well.

For sources take a look at

1. The book: *Classical and Multilinear Harmonic Analysis*, Camil Muscalu,
2. The Wikipedia article: https://en.wikipedia.org/wiki/Fourier_transform#On_Lp_spaces

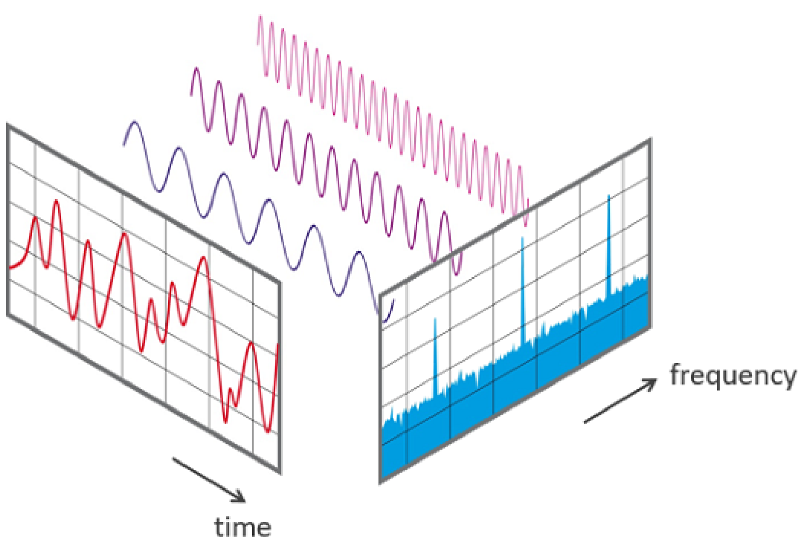

Figure 1: A depiction of a function (of time) and the frequencies it consists of.