MAT2000 Project:

The uniqueness and existence of solutions to nonlinear PDEs

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Nonlinear partial differential equations (PDEs) are the mathematical formulations of physical laws. And like the laws of physics, separate PDEs can create very different phenomena. The study of a particular PDE may require specially tailored mathematical tools (there is in fact no general theory for nonlinear PDEs).

This project is devoted to the study of *hyperbolic conservation laws*

$$u_t + \mathrm{div}_x f(u) = 0,$$

where $u(x,t)\colon \mathbb{R}^d \times \mathbb{R}_+ \to \mathbb{R}$ is the unknown density of mass and $f\colon \mathbb{R} \to \mathbb{R}^d$ is a given flux function (which determines the 'physics'). Such PDEs serve as simple models for the flow of fluids, traffic, and crowds. A typical 'nasty' feature of such equations is the development of discontinuities (like a sudden traffic cork) where the function u is no longer differentiable (or continuous) rendering the original PDE meaningless. The project will develop the theory of entropy solutions that overcomes the discontinuity-issue by re-interpreting the equation in an energy-preserving sense. The mathematics will include the method of characteristics, elliptic regularization, and the classical existence and uniqueness proofs for entropy solutions of hyperbolic conservation laws.

For sources take a look at

1. The book: *Front Tracking for Hyperbolic Conservation Laws*, Helge Holden, Nils Henrik Risebro,
2. The Wikipedia articles: https://en.wikipedia.org/wiki/Burgers%27_equation
3. and https://en.wikipedia.org/wiki/Conservation_law

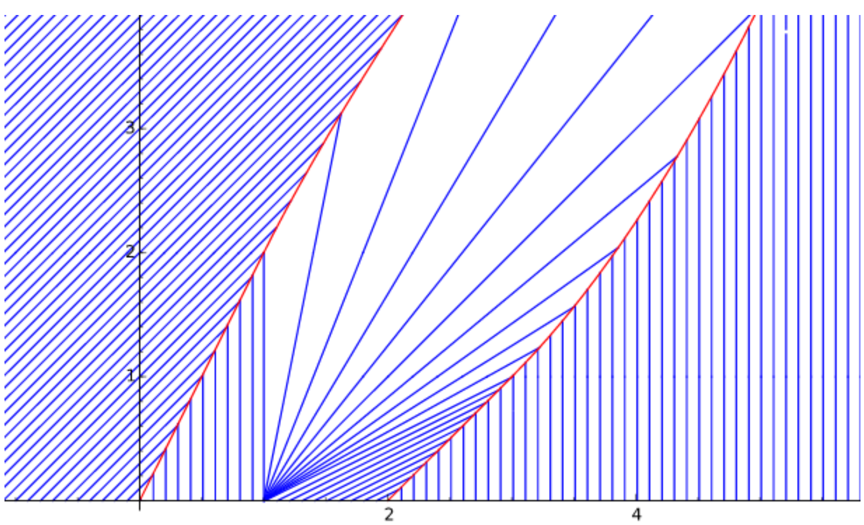

Figure 1: A space-time diagram of a solution u solved using the method of characteristics