Enumerative Geometry: Hurwitz Numbers

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Enumerative geometry

In its most pure form, the goal of *enumerative geometry* is to count the number of solutions to geometric problems. An early example of this is the Problem of Apollonius: Given three circles in the plane, how many circles can be constructed that tangent to all three?

Important and surprising discoveries in the 1990s revealed that many classical counting problems had fundamental relationships with high-energy physics, particularly string theory. This lead to a revolution in the field, and goals of modern enumerative geometry are much broader than those of its roots. Indeed, modern enumerative geometry has aspects dealing with geometry, number theory, combinatorics, algebra, physics and representation theory.

Hurwitz numbers

Hurwitz numbers are a classical counting theory dating back to Hurwitz in 1987. Hurwitz introduced these numbers as counts of special objects in complex analysis. However (like most aspects of enumerative geometry) the numbers were found to be interesting objects to study other areas, such as combinatorics, physics and algebraic geometry. Indeed, these numbers can be studied entirely within these individual fields. Hurwitz numbers continue to be studied to this day, and more and more surprising properties are found all the time.

About the project

The project is a study of Hurwitz numbers. This can be undertaken from many different aspects, depending on the interests and background of the student. For example if the student is interested in combinatorics, then the project can be undertaken from a combinatorial viewpoint. The same applies for complex analyis, algebraic geometry or algebra/group theory.