Enumerative Geometry: Box Stacking

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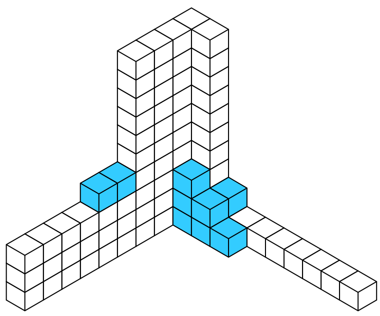
Enumerative geometry

In its most pure form, the goal of *enumerative geometry* is to count the number of solutions to geometric problems. An early example of this is the Problem of Apollonius: Given three circles in the plane, how many circles can be constructed that tangent to all three?

Important and surprising discoveries in the 1990s revealed that many classical counting problems had fundamental relationships with high-energy physics, particularly string theory. This lead to a revolution in the field, and goals of modern enumerative geometry are much broader than those of its roots. Indeed, modern enumerative geometry has aspects dealing with geometry, number theory, combinatorics, algebra, physics and representation theory.

Stacked boxes and melting crystals

A particularly interesting counting problem, which has become very important in recent years, relates to stacked boxes. Simply stated, we are interested in counting the number of ways to stack boxes in a corner. However, not all corners are the same and complexity arises when the corners have different shapes as seen in the diagram below:



Counting such configurations is phyiscally interesting and can be motivated by the melting of crystals: *"Adding boxes"* becomes *"removing parts of the crystal"*.

About the project

The project is a study of the counting of stacked boxes. This can be undertaken from many different aspects, depending on the interests and background of the student. For example if the student is interested in combinatorics, then the project can be undertaken from a combinatorial viewpoint. The same applies for algebraic geometry or algebra/group theory. The project is particularly interesting when studied using operator algebra.