

MAT2000 Project:

Growth rate of groups

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One of the most basic notions in mathematics is that of a group. In an introductory course to algebra (for instance MAT2200) you have probably seen some useful tools to study *finite* groups. In this project we will get familiar with some of the techniques and results in geometric group theory, which uses geometric techniques to study *infinite* groups that are finitely generated.

We will focus on an invariant from geometric group theory known as the growth function γ_G of a group G . It is defined as follows: Letting S be a finite generating set for G , $\gamma_G(n)$ is the number of elements $g \in G$ which can be written as a product $g = s_1 \cdots s_k$ where $s_i \in S$ and $k \leq n$. Infinite groups can be divided into different classes based on the asymptotic behavior of their growth function. For instance, if γ_G grows at most like a polynomial function, then G is said to have *polynomial growth*.

A natural first goal of this project is to prove that finitely generated, virtually nilpotent groups have polynomial growth. Gromov's famous theorem states that the converse is also true. Another slightly more involved goal is to prove the intermediate growth of certain specific groups, like the first Grigorchuk group. There are several possible directions in this project, feel free to contact me if this sounds interesting to you. To learn a bit more about geometric group theory, see for instance the introduction to the book of Drutu and Kapovich which is freely available online.

Prerequisites: You should have taken a course in abstract algebra, e.g. MAT2200 Groups, rings and fields.

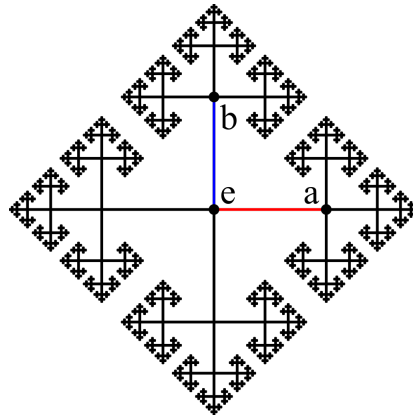


Figure 1: The Cayley graph of the free group on two generators a and b .