MAT2000 Project:
Sampling of bandlimited functions

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Say you have a continuous-time signal that you want to represent discretely by sampling it along a set of equally spaced points. How often do you have to sample in order to be able to fully reconstruct your signal from the samples? This basic question of signal processing is answered by the sampling theorem of Nyquist–Shannon–Whittaker: The sampling rate must be more than twice the size of the highest frequency in your signal.

Mathematically, the sampling theorem can be viewed as a statement about Paley–Wiener spaces, which are important function spaces in harmonic analysis. The Paley–Wiener space $PW_a(\mathbb{R})$ consists of complex-valued functions on $\mathbb{R}$ whose Fourier transform is supported within the interval $[-a, a]$. Such functions are often called bandlimited. In this context, the sampling theorem states that every $f \in PW_a(\mathbb{R})$ is determined (in a stable way) by its values on the set $b\mathbb{Z} = \{\dots, -b, 0, b, 2b, \dots\}$ whenever $b < 1/(2a)$.

This project revolves around sampling of functions in the Paley–Wiener space. A first goal of this project is to give a proof of the sampling theorem. Then one can delve into additional topics such as irregular sampling, that is, when the samples are not equally spaced. There are several possible directions; do not hesitate to contact me if you have questions or suggestions.

Prerequisites: You should have at least one course in rigorous analysis, for example in the form of MAT2400 (Real analysis) and/or MAT2410 (Introduction to Complex Analysis). I also recommend taking MAT3400/4400 (Linear analysis with applications) parallel to writing the project.

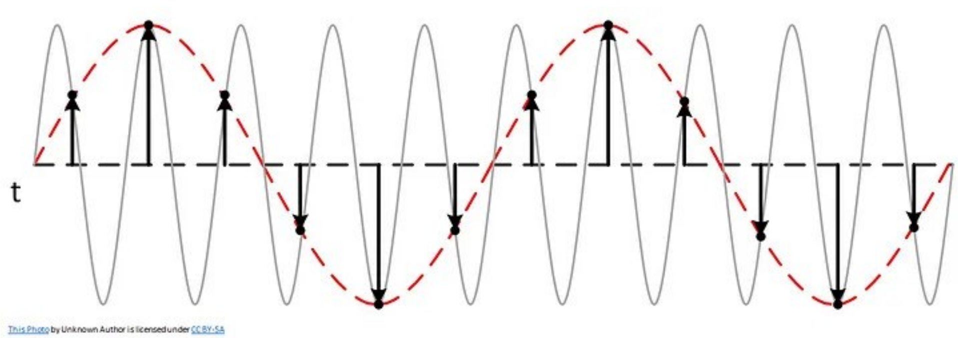

Figure 1: The grey signal agrees with the red dashed signal on the given sampling set. A higher sampling rate is needed to determine the grey signal.