MAT2000: Topological invariants of 2-dimensional Unoriented manifolds

Supervision: David Jaklitsch

Motivation: Topological quantum field theories are mathematical objects that assign scalar invariants to manifolds in a way compatible with cutting manifolds and gluing them at a common boundary. They relate geometry and algebra, but are also of interest in theoretical physics (describe the fractional quantum Hall effect and Chern-Simons theory).

A *folkore* theorem states that 2-dimensional topological field theories for oriented manifolds correspond to *commutative Frobenius algebras*. If, instead, one considers *unoriented manifolds*, i.e., manifolds without a choice of orientation and non-orientable manifolds, the algebraic object necessary to construct such a topological field theory requires an additional structure, namely an involution.

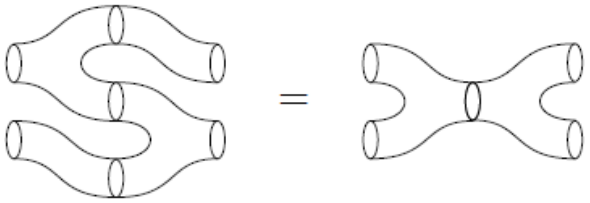

Figure 1: Topological invariance of the values assigned to glued manifolds

Goal: In a first stage the student should learn generalities about *manifolds and cobordisms* and the definition of *Frobenius algebras* and $*$*-algebras*. Afterwards, the idea is to study one or two of the main constructions of two-dimensional topological field theories for unoriented manifolds: in [KM97] are described the so-called *lattice* topological field theories and in [SchP12] the *fully extended* TFT's for unoriented manifolds are classified by a type of $*$*-Frobenius algebras*. Depending on progress, in an advanced stage of the project, it would be interesting to make a connection between these two approaches and compare computations for specific examples.

References

[links] `https://en.wikipedia.org/wiki/Frobenius_algebra`
`https://ncatlab.org/nlab/show/2d+TQFT`

[FHK94] M. Fukuma, S. Hosono, H. Kawai, *Lattice topological field theory in two dimensions*, Communications in Mathematical Physics vol 161, pages 157–175 (1994)

[KM97] V. Karimipour, A. Mostafazadeh, *Lattice topological field theory on nonorientable surfaces*, J. Math. Phys. 38, 49, (1997).

[SchP12] C. Schommer-Pries, *The Classification of Two-Dimensional Extended Topological Field Theories*, `math.AT/1112.1000`, (2012).

[TVir17] V. Turaev, A. Virelizier, *Monoidal Categories and Topological Field Theory*. Springer, Progress in Mathematics series, Volume 322, (2017).