25th February, 2021

MAT2200

Mandatory assignment 1 of 1

Submission deadline

Thursday 11th MARCH 2021, 14:30 in Canvas (canvas.uio.no).

Instructions

You can choose between scanning handwritten notes or typing the solution directly on a computer (for instance with LaTeX). The assignment must be submitted as a single PDF file. Scanned pages must be clearly legible. The submission must contain your name, course and assignment number.

It is expected that you give a clear presentation with all necessary explanations. Remember to include all relevant plots and figures. Students who fail the assignment, but have made a genuine effort at solving the exercises, are given a second attempt at revising their answers. All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, we may request that you give an oral account.

In exercises where you are asked to write a computer program, you need to hand in the code along with the rest of the assignment. It is important that the submitted program contains a trial run, so that it is easy to see the result of the code.

Application for postponed delivery

If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) well before the deadline.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination.

Complete guidelines about delivery of mandatory assignments:

uio.no/english/studies/admin/compulsory-activities/mn-math-mandatory.html

GOOD LUCK!

To pass the assignment, you need a score of 60 %.

Problem 1. (a) (10 points) Draw the poset diagram for subgroups of the cyclic group of order 30.

(b) (10 points) How many elements of orders $2, 3, 4$ does the direct sum group $\mathbb{Z}_{30} \oplus \mathbb{Z}_2$ have?

(c) (10 points) Draw the poset diagram for subgroups of $\mathbb{Z}_{15}^*$ under multiplication mod(15).

Problem 2. (10 points) Let G be a group and suppose that H is a finite subgroup of G with $|H| = n$ for some $n \in \mathbb{Z}^+$. Show that if H is the only subgroup of G of order n, then it is normal.

Problem 3. (30 points) Let S_4 be the symmetric group on the set $\{1, 2, 3, 4\}$ and let A_4 be the alternating group.

(a) Refer to Example on page 87 in A. Terras' book. Write down an explicit identification of A_4 with the group of proper rotations, meaning orientation-preserving and size-preserving, of a tetrahedron in 3-space. More precisely, number the vertices of the tetrahedron by $1, 2, 3, 4$ and indicate what permutations of the vertices, seen as elements in A_4, correspond to the various proper rotations of the tetrahedron. Describe the rotations by specifying the axis of rotation and the angle of rotation. Show that A_4 has generators the cycle $R = (234)$ in S_4 and the product of transpositions $F = (12)(34)$ in S_4, with relations $R^3 = F^2 = (FR)^3 = I$.

(b) Find all the conjugacy classes in A_4. You may use without proof the general fact valid in S_n that cycles are conjugate into cycles of the same length, more precisely that $\sigma(j_1 j_2 \ldots j_k)\sigma^{-1} = (\sigma(j_1)\sigma(j_2)\ldots\sigma(j_k))$ for every $\sigma \in S_n$ and every cycle $(j_1 j_2 \ldots j_k)$ in S_n.

(c) Using part (b) of this problem, write the class equation of A_4. Show that A_4 has a unique normal subgroup H, with H of order 4. (Hint: Recall that for a subgroup H to be normal in a group G we must have that for every $h \in H$, its conjugates ghg^{-1} are in H for all $g \in G$. Alternatively, use Problem 2.) Show that A_4 does not have a subgroup of order 6. (This shows that the converse of Lagrange's theorem does not hold in general.)

Problem 4. (10 points) Let $G = \mathbb{Z}_{36}$ and consider the subgroups $H = \langle 18 \rangle$ and $K = \langle 9 \rangle$.

(a) List the cosets in G/H, G/K and K/H. For each coset, give all the elements. For example, note that $0 + H = \{0, 18\}$ in G/H,

$0 + K = \{0, 9, 18, 27\}$ in G/K and $0 + H = \{0, 18\}$ in K/H. There are 17 more cosets in G/H, 8 more cosets in G/K, and one more coset in K/H.

(b) Write explicitly the bijection from the cosets in G/K to the cosets in $(G/H)/(K/H)$ prescribed by the third isomorphism theorem (Exercise 3.5.9 in A. Terras' book).

Problem 5. (20 points)

(a) Let G be a group and A, B be normal subgroups of finite index, thus $(G : A) = |G/A|$ and $(G : B) = |G/B|$ are both finite. Show that $(G : A \cap B)$ is finite and

$$(G : A \cap B) \leq (G : A)(G : B).$$

(b) Show that the inequality above is valid without assuming that A, B are normal subgroups. (Hint: Find an injective map from the collection of cosets in $B/(A \cap B)$ to the collection of cosets in G/A, similar to the proof of the second isomorphism theorem.)