1) Binary operation $a*b = \max\{a,b\}$ is $a \in \mathbb{R}$.

Commutative: $\forall\, a,b \in \mathbb{R}$ $\quad a*b = b*a$ $\quad$ commutative ✓

$$a*b = \max\{a,b\} = \max\{b,a\} = b*a$$

associative $\forall\, a,b,c \in \mathbb{R}$ $\quad a*(b*c) = (a*b)*c$

$$\max\{a, \max\{b,c\}\} = \max\{a,b,c\} = \max\{\max\{a,b\}, c\}$$

$\Rightarrow$ associative ✓

2) Is $a*b = a/b$ a binary operation on $\mathbb{Q}$?

$*: S \times S \to S$ $\qquad$ No! since $b=0$ $\quad a*b \notin \mathbb{Q}$.

not a binary operation $\quad$ is commutative? NO

Take instead $(\mathbb{Q}^\times, *)$ $\qquad$ Ques. is associative? NO

3) $(S, *)$ $(T, \#)$ binary structures a bijection $f: S \to T$ is an isomorphism if

$$\underbrace{f(\overbrace{x * y}^{\in S})}_{\in T} = \underbrace{f(\overbrace{x}^{\in S})}_{\in T} \# \underbrace{f(\overbrace{y}^{\in S})}_{\in T} \qquad x, y \in S.$$

4) $S = \mathbb{Z}$ $\quad T = \{2n \mid n \in \mathbb{Z}\}$ both with $+$.

$(S, +) \cong (T, +)$?

Step 1 Is there a bijection between S and T? $\ni m$

$\cancel{f(n) = n+2}$

$\cancel{f(1) = 1+2 = 3 \notin T}$

$f(n) = 2n$

$f: S \to T$ is a bijection $f^{-1}(m) = \frac{m}{2}$.

$\forall\ n, m \in S$

$$f(n+m) = 2(n+m) = \underset{\substack{\parallel \\ f(n)}}{2n} + \underset{\substack{\parallel \\ f(m)}}{2m}$$

$$= f(n) + f(m).$$

Are $(S, *)$ and $(S', *')$ isomorphic?

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Check list:

1) Do S and S' have same cardinality?

$|S| \in \mathbb{N}$ size if S is finite

Ex. No isomorphism between $(\mathbb{Q}^{\times}, \cdot)$ and $(\mathbb{R}^{\times}, \cdot)$

2) Do $(S, *)$ and $(S', *')$ have identity elements?

Thm 3.~~1~~8 missing

3) Order of elements (to come in section 5)