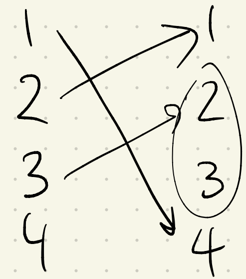
Quiz

1) size of symmetric group on $\{1, \ldots, 4\}$ "S_4"

$$|S_n| = n!$$



$$4 \cdot 3 \cdot 2 \cdot 1 = 4! = 24$$

2. $\sigma \in S_6$ $\quad \sigma(i) := 1 +_6 i$

$\sigma(1) = 2$

$\sigma(2) = 3$

$\sigma(5) = 5+1 = 6$

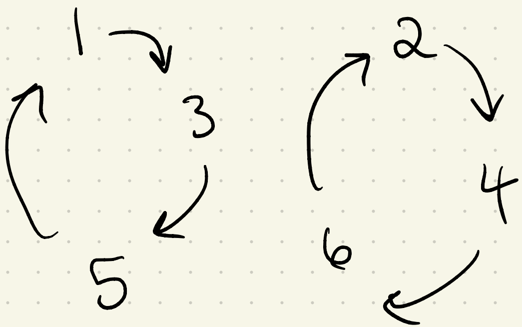
$\sigma(6) = 6+1$

$1 \to 2 \to 3 \to 4 \to 5 \to 6 \to 1$

σ has one orbit of size 6.

$\Rightarrow \sigma$ is a cycle of size 6.

3. $\tau \in S_6$.



$$\tau(1) = 1+2 = 3$$
$$\tau(2) = 2+2 = 4$$
$$\vdots$$
$$\tau(5) = 5+2 = 1$$
$$\tau(6) = 6+2 = 2.$$

$\underline{\tau = (1,3,5)(2,4,6)}$ in cycle notation

4. Which perms in S_6 are equal (in cycle notation)

a) $(1,2,3)(4,5,6) = (3,1,2)(4,5,6)$

b) not the same

c) not the same $(1,2,3) \neq (1,3,2)$

d) same $(1) = e$

From video:

"S_A is not abelian for $|A| \geq 3$"

$|A| = 3$ <u>Find</u>: σ_1, σ_2 s.t. $\sigma_1\sigma_2 \neq \sigma_2\sigma_1$.

If σ_1, σ_2 are cycles they cannot be disjoint.

$\sigma_1 = (1, 2, 3)$

$\sigma_2 = (1, 3)\cancel{(2)}$

$\sigma_1\sigma_2(1) = \sigma_1(3) = 1$

$\sigma_2\sigma_1(1) = \sigma_2(2) = 2$.

permutations with at most one orbit of size > 1. $\Rightarrow$ Cycles on $\{1, \dots, n\}$ $\underline{\subseteq}$ subset. S_n $\Leftarrow$ permutations on n.

A <u>cycle</u> has a largest orbit (unique orbit of size >1).

σ_1, σ_2 cycles are disjoint if their unique orbits of size >1 don't intersect

$B_1 \cap B_2 = \emptyset$.

Def 5.3 The <u>order</u> of a group G is $|G|$.

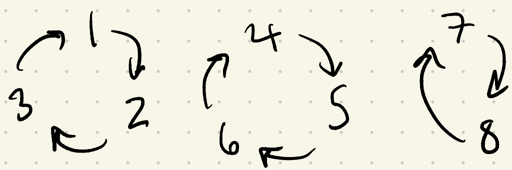
* The <u>order</u> of $a \in G$ is $|\langle a \rangle|$ where $\langle a \rangle \leq G$. Write $\mathrm{ord}(G)$ or $\mathrm{ord}(a)$.

- If $\mathrm{ord}(a) = k$ then $a^k = e$. $\left(a^l \neq e \text{ for } 0 < l < k\right)$
- Next week: If $a^r = e$ then $\mathrm{ord}(a) \mid r$.
  Lagrange's Thm.
- If $\varphi : G \to G'$ is an isomorphism, then $\mathrm{ord}(g) = \mathrm{ord}(\varphi(g))$ $\forall\, g \in G$.

Question What are the orders of the following permutations? $6 \in S_8$ Hint draw cycle diagram.

1) $(1,2,3)(4,5,6)(7,8)$

6



2) $(1,2,3)(4,5)(7)(8)$

6

3) $(1,2,3,4)(5,6)(7,8)$

4

4) $(1,2)(3,4)(5,6)(7,8)$

4 2? not small enough. $= 2$

5) $(1,2,3,4,5)(6,7,8)$

~~5 × 2.~~ 15.

6) $(1,2,3,4,5,6,7,8)$

8.

Thm The order of a permutation σ is

$$\mathrm{lcm}(|B_1|, \ldots, |B_r|)$$ where $B_1, \ldots, B_r$

"least common multiple"

are the orbits of σ.

Exercise consider $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 1 & 3 \end{pmatrix} \in S_5$.

Let $H = \langle \sigma \rangle \leq S_5$

- write σ in cycle notation and determine $\operatorname{ord}(\sigma)$
- using $\sigma = \mu_1 \cdots \mu_r$ cycle notation determine all powers of σ.
- Is $H \cong S_n$ for some n?