Quiz

$\langle (1,1,1) \rangle$

$\parallel$

1) $\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5$ $\gcd(2,3) = \gcd(3,5) = \gcd(2,5) = 1$

$\overset{\text{Cor } 11.6}{\cong} \mathbb{Z}_{30}$

$\cong \mathbb{Z}_2 \times \mathbb{Z}_{15} \cong \mathbb{Z}_3 \times \mathbb{Z}_{10} \cong \mathbb{Z}_5 \times \mathbb{Z}_6$

symmetric group on 3 elts.

2) The order of $S_3 \times \mathbb{Z}_{10}$ is. $|S_3 \times \mathbb{Z}_{10}|$

$= |S_3||\mathbb{Z}_{10}| = 3! \cdot 10 = 60.$

$\leftarrow \mathbb{Z}$ with addition.

3) The map $f: \mathbb{Z} \to \mathbb{Z}$ given by $f(n) = n+1$

Thm 13.12 part 1. If $f: G \to G'$ is a homom. then

$f(e) = e'$ $e = 0 \in \mathbb{Z}$ but $f(0) = 1 \neq e$. f is not a homom.

4) If $f: G \to G'$ is a homomorphism then f need not be injective, surjective nor an isomorphism.

Counter example:

Trivial homomo $f: G \to G'$ sends $f(g) = e' \; \forall g$

- If $|G| > 1$ then f is not injective
- If $|G'| > 1$ then f is surjective

In either case then f is not an isomorphism.

By Thm 13.2: A homom satisfies $f(e) = e'$ and $(f(a))^{-1} = f(a^{-1}) \; \forall \; a \in G$.

Review of order:

Def 5.3 The order of a group G is $|G| = \text{ord}(G)$.

* The order of $a \in G$ is $|\langle a \rangle|$ where $\langle a \rangle \leq G$. Write $\text{ord}(G)$ or $\text{ord}(a)$.

- If $\text{ord}(a) = k$ then $a^k = e = a^0$ $\left(a^l \neq e \text{ for } 0 < l < k\right)$
- Next week: If $a^r = e$ then $\text{ord}(a) \mid r$.
  Lagrange's Thm.
- If $\varphi : G \to G'$ is an isomorphism, then $\text{ord}(g) = \text{ord}(\varphi(g))$ $\forall\, g \in G$

From direct product. if $\nexists$ an element $a \in G$ with $\operatorname{ord}(a) = |\langle a \rangle| = |G|$ then G is not <u>cyclic</u> since a <u>cyclic group</u> (see pg 59) is a group G s.t.

$$G = \{ a^n = \underbrace{a * \ldots * a}_{n \text{ times}} \mid n \in \mathbb{Z} \} \text{ for some } a \in \mathbb{Z}.$$

Used this to see that $\mathbb{Z}_3 \times \mathbb{Z}_3$ is not cyclic since $|\langle (a,b) \rangle| \leq 3 < 9 = |\mathbb{Z}_3 \times \mathbb{Z}_3|$

$\mathbb{Z}_3 = \{0, 1, 2\}, +_3$

$$\boxed{|\langle (0,0) \rangle| = 1}$$

$\underset{\{0,1\}}{\underset{\parallel}{\mathbb{Z}_2}} \times \underset{\{0,1,2\}}{\underset{\parallel}{\mathbb{Z}_3}}$ is a cyclic group.

$$\begin{aligned}
(1,1) &= (1,1)\\
2(1,1) &= (1,1)+(1,1) = (0,2)\\
3(1,1) &= (1,1)+(1,1)+(1,1) = (1,0)\\
4(1,1) &= 3(1,1)+(1,1) = (1,0)+(1,1) = (0,1)\\
5(1,1) &= 4(1,1)+(1,1) = (0,1)+(1,1) = (1,2)\\
6(1,1) &= 5(1,1)+(1,1) = (1,2)+(1,1) = (0,0) = e.
\end{aligned}$$

$(1,1)$ has order 6 in $\mathbb{Z}_2 \times \mathbb{Z}_3$

$\langle (1,1) \rangle = \mathbb{Z}_2 \times \mathbb{Z}_3$

is cyclic

$\Longrightarrow \mathbb{Z}_2 \times \mathbb{Z}_3 \cong \mathbb{Z}_6.$

1) $G = \mathbb{Z}_2 \times \mathbb{Z}_3$ write down (left or ~~right~~) cosets of $H = \langle (1,0) \rangle$.

$a + H = \{ \quad\underline{\qquad\qquad}\quad \}$

2) $G = \mathbb{Z}_2 \times \mathbb{Z}_3$ write down ____ cosets of $H = \langle (0,1) \rangle$.

3) $G = \mathbb{Z}_3 \times \mathbb{Z}_3$ write down ____ cosets of $H = \langle (1,1) \rangle$

4) $G = \mathbb{Z}_3 \times \mathbb{Z}_3$ write down ____ of $H = \langle (1,0) \rangle$.

1) $G = \mathbb{Z}_2 \times \mathbb{Z}_3$ write down (left or right) cosets of $H = \langle (1,0) \rangle$.

$a + H = \{ \underline{\qquad\qquad} \}$

$|G| = 6 \qquad |H| = 2 \implies [G:H] = \#\text{ cosets} = \frac{|G|}{|H|}$

$= 3$.

$H = \{ \underline{\qquad\qquad} \}$

$(0,1) + H = \{ \underline{\qquad\qquad} \}$

$(0,2) + H = \{ \underline{\qquad\qquad} \}$

2) $G = \mathbb{Z}_2 \times \mathbb{Z}_3$ write down cosets of $H = \langle (0,1) \rangle$.

$|G| = 6 \qquad |H| = 3 \Rightarrow [G:H] = \underline{2}$

$H = \{(0,0), (0,1), (0,2)\}$

$(1,0) + H = \{(1,0), (1,1), (1,2)\}$

3) $G = \mathbb{Z}_3 \times \mathbb{Z}_3$ write down cosets of $H = \langle (1,1) \rangle$

2) $G = \mathbb{Z}_2^{+_2} \times \mathbb{Z}_3^{+_3}$, $\{0,1,2\}$ write down

cosets of $H = \langle (0,1) \rangle$.

$|G| = 6$

$(0 +_2 0, 1 +_3 1)$ $\quad H' = \langle (1,1) \rangle$ Ex 11.3

$H = \{ (0,0), (0,1), (0,2) \}$ $\quad (1,1) + (1,1) := (1 +_2 1, 1 +_3 1)$

$= (0,2)$

$|H| = 3$

$[G : H] = 2$

$(1,0) + H = \{ \quad \}$

$(1,2) + H = \{ \quad \}$