$$X = C([0,1], \mathbb{R}), \quad \|f\| = \sup_{x \in [0,1]} |f(x)|.$$

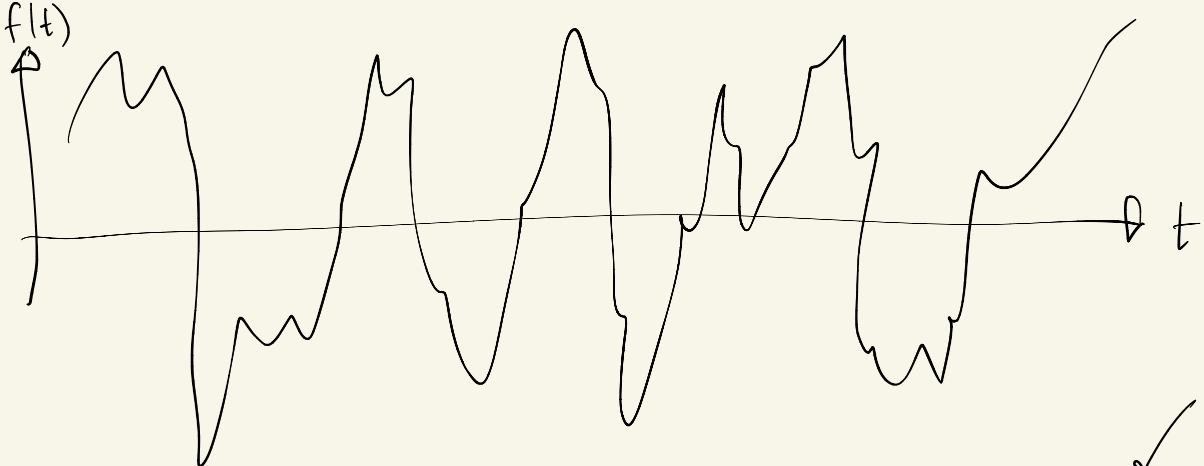
$$T(f)(x) = \int_0^x g(f(y))\,dy, \quad g \text{ er Lipschitz med konst. } 1/2$$

$$|T(f)(x) - T(h)(x)| \leq \int_0^x |g(f(y)) - g(h(y))|\,dy$$

$$\leq \frac{1}{2} \int_0^x \underbrace{|f(y) - h(y)|}_{\leq \|f-h\|}\,dy$$

$$\leq \frac{1}{2}\|f-h\| \int_0^x 1\,dy = \frac{1}{2}\|f-h\|\,x$$

$$\Rightarrow \quad \|T(f) - T(h)\| \leq \sup_x \frac{1}{2}\|f-h\|\,x = \frac{1}{2}\|f-h\|$$


f(t)
t


sin(t)
sin(10·t)

$$F'(x;r) = \lim_{h\to 0} \frac{F(x+hr) - F(x)}{h} \qquad F: X \to Y$$

$$\Longleftrightarrow \quad \left\| \frac{F(x+hr)-F(x)}{h} - F'(x;r) \right\|_Y \xrightarrow[h\to 0]{} 0$$

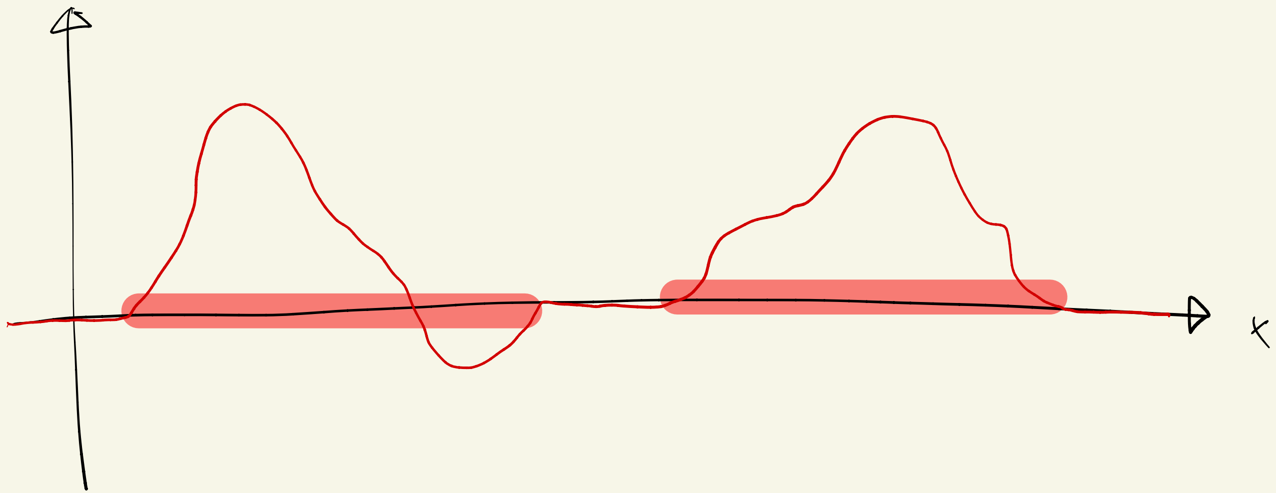
La $\quad A(r) = F'(x;r)$

$$\frac{\| F(x+r) - F(x) - A(r) \|_Y}{\|r\|_X} \xrightarrow[r\to 0]{} 0$$

$A : \mathbb{R}^n \to \mathbb{R}^m$ linear, $n \neq m$ : da $\exists\ M \in \mathbb{R}^{m \times n}$ s.a.

$$A(x) = Mx \qquad \forall\ x$$

$A : V \to W$, $\quad W \subsetneq V$, $\quad A$ invertibel

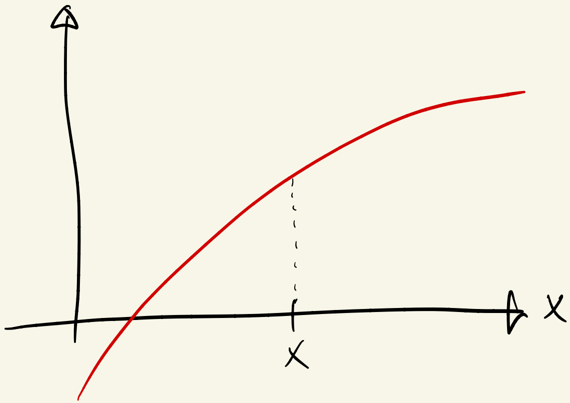

x

$B(X,Y) = \{ \text{begrensede } f : X \to Y \}$,

$d(f,g) = \sup_x d_Y(f(x), g(x))$

- Hvis $Y$ er komplett, er $B(X,Y)$ komplett.
- $C_b(X,Y)$ er komplett når $Y$ er komplett

$$\|f-g\|_{L^1} = \int_0^1 |f(x)-g(x)|\,dx$$



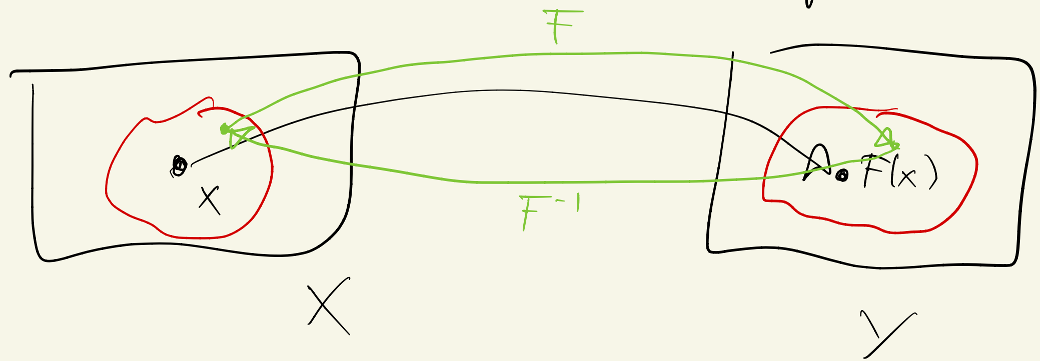

$f'(x) \neq 0$

$f'$ er kont. i $x$

$F : X \to Y$, $\quad F'(x)$ er inverterbar, $F'$ kont. i $x$

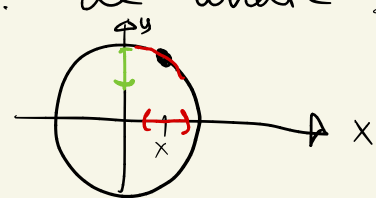
Banach's lemma: hvis $A \in \mathcal{L}(X, Y)$ inverterbar $\exists\, r > 0$ s.a. alle $B \in \mathcal{L}(X, Y)$, $\|A - B\|_{\mathcal{L}} < r$ er inverterbare

Invers funk. thm.: $F : X \to Y$, er $F$ bijektiv i en omegn om et punkt $x \in X$?




Implisitt funk. teorem: Gitt et likningssett med flere variable, kan du løse for én av dem mhp. de andre?

$x^2 + y^2 = 1$

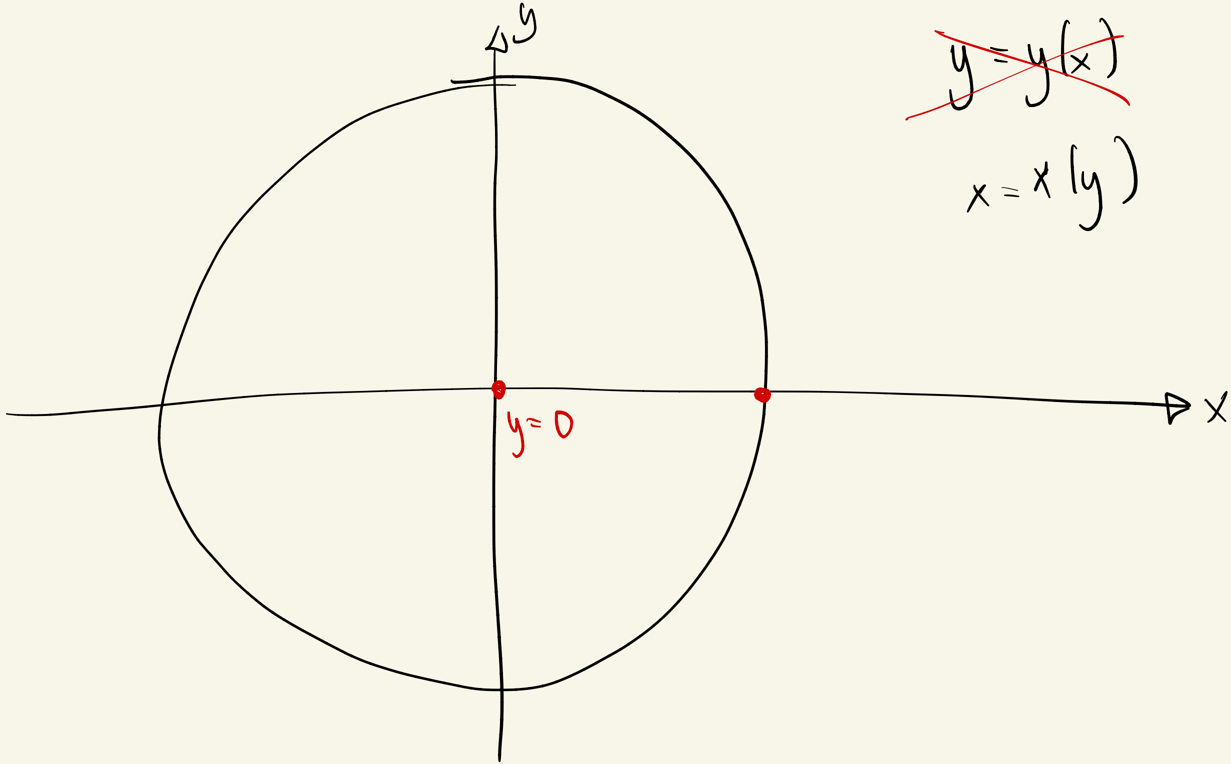


$$F : \mathbb{R}^3 \to \mathbb{R}^2$$

$F = F(x, y, z)$, $\qquad F(x, y, z) = 0$

Oppg: Kan du finne funk. $X = X(z)$, $Y = Y(z)$ s.a.

$$F(X(z), Y(z), z) = 0$$




$\cancel{y = y(x)}$

$x = x(y)$