UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Examination in MAT-INF1310 — Ordinary Differential Equations

Day of examination: June 11, 2009

Examination hours: 9.00 – 12.00

This problem set consists of 2 pages.

Appendices: None

Permitted aids: None

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1 (weight 25%)

Consider the third-order differential equation

$$y^{(3)}(x) = 3y'(x) - 2y(x) + 12e^x. \tag{1}$$

a

Find all solutions of the homogeneous equation associated to (1).

b

Find a particular solution of (1).

c

Find a solution of (1) which satisfies the initial value condition $y(0) = 2$.

Problem 2 (weight 20%)

The function $y(x) = e^x$ is the unique solution of the initial value problem

$$\frac{dy}{dx} = y, \qquad y(0) = 1. \tag{2}$$

a

Approximate the values e^2 and e^5 by applying Euler's method with step size $h = 1$ to the initial value problem (2).

b

Let n be any positive integer. Approximate the value e^n by applying Euler's method with step size $h = 1$ to the initial value problem (2).
(*Use induction if necessary.*)

(Continued on page 2.)

Problem 3 (weight 25%)

Let I be an open (non-empty) interval and suppose that the functions p, q, and f are continuous on I. Consider the second-order differential equation

$$y''(x) + p(x)y'(x) + q(x)y(x) = f(x), \qquad x \in I. \tag{3}$$

Recall that a complementary function of (3) is a solution of the homogeneous equation associated to (3).

a

Show that the complementary functions of (3) form a real vector space, i.e., show that if $y_1(x)$ and $y_2(x)$ are complementary functions then the function

$$Y(x) = c_1 y_1(x) + c_2 y_2(x), \qquad x \in I,$$

is a complementary function for all real numbers c_1 and c_2.

b

Show that if $y_p(x)$ is a solution of the differential equation (3) and $y_c(x)$ is a complementary function of (3), then the function

$$Y(x) = y_p(x) + y_c(x), \qquad x \in I,$$

is a solution of the differential equation (3).

c

Suppose that $y_p(x)$ is a solution of the differential equation (3). Show that if $Y(x)$ is another solution of the same equation, then there exists a complementary function $y_c(x)$ such that

$$Y(x) = y_p(x) + y_c(x), \qquad x \in I.$$

Problem 4 (weight 30%)

Consider the system

$$\begin{cases} \dfrac{dx}{dt} = x - 5y + 2\sin t \\ \dfrac{dy}{dt} = x - y - 3\cos t \end{cases} \tag{4}$$

of first-order differential equations.

a

Find all solutions of the homogeneous system associated to (4).

b

Find a particular solution of (4).