Exercises

Ex. 2 p. 374 (from SSS' book)

We consider the problem

$$\max \int_0^1 (1 - x^2 - u^2)\, dt, \quad \dot{x} = u, \quad x(0) = 0, \quad , \quad x(1) \geq 1, \quad u \in (-\infty, \infty)$$

Write down the maximum principle theorem for this problem and find the optimal solution.

Ex. 9 p. 375 (from SSS' book)

We consider the problem

$$\max \int_0^2 (x^2 - u)\, dt, \quad \dot{x} = u, \quad x(0) = 1, \quad x(2) \text{ free}, \quad u \in [0, 1].$$

Write down the maximum principle theorem for this problem. Show that $p(t)$ is strictly decreasing and find the optimal solution.

Exercise (from exam 2009)

We consider the problem

$$\max \int_0^{\frac{\pi}{6}} \left(u(t) - \frac{u(t)^2}{\cos(t)} \right) dt$$

where

$$\begin{cases} \dot{x} = -u, \\ u(t) \geq 0, \\ x(0) = 0, \\ x(\frac{\pi}{6}) = -\frac{1}{8}. \end{cases}$$

You can take for granted that this is a standard problem.

a) Write down the Hamiltonian function $H(t, x, u, p)$ for this control problem. Apply the maximum principle and show that the adjoint function $p(t)$ must be equal to a constant that we denote k.

b) Find the only possible control function u^* for this problem as a function of k and other known functions.

c) Determine the constant k and the optimal pair (x^*, u^*). Justify the fact that this pair is indeed optimal.

d) We change the terminal conditions to $x(0) = 0$ and $x(\frac{\pi}{6}) \geq -\frac{1}{8}$. What is the solution of the new control problem?