The goal is to plot the solution in the phase-plane of the system

$$\begin{cases} \dfrac{dx}{dt} = x^2 - 2xy \\ \dfrac{dy}{dt} = 2xy - y^2. \end{cases}$$

Let $X(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$, we denote $\frac{dX}{dt} = F(X)$ where

$$F(X) = \begin{pmatrix} x^2 - 2xy \\ 2xy - y^2 \end{pmatrix}.$$

a) Find the equilibrium and the vertical and horizontal isoclines.

b) How does the vector field look like on the lines $y = x$ and $y = -x$?

c) We consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}.$$

What does this transformation represent geometrically? Show that

$$F \circ T = -T \circ F.$$

How can we use this result for plotting the vector field?

d) We consider the linear transformation $\bar{T} : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$\bar{T} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ -x \end{pmatrix}.$$

What does this transformation represent geometrically? Show that

$$F \circ \bar{T} = \bar{T} \circ F.$$

How can we use this result for plotting the vector field?

e) Plot the vector field and some solution curves in the phase plan.