Obligatory exercise for MAT2440
Spring 2015

Deadline: Wednesday 15 April.

All the problems can be solved without the aid of a computer or calculator. However you may also use any computer system, like Matlab, Mathematica, Maple, or the numpy package of Python. Whenever doing so, you should document the commands you are using and explain in detail how you apply the results.

Problem 1

Let

$$P(x,y) = 5x^4y - y^5, \qquad \text{and} \qquad Q(x,y) = x^5 - 5xy^4.$$

Solve the differential equation

$$Pdx + Qdy = 0$$

as an exact equation.

Problem 2

Let

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}.$$

Find a general solution to the differential equation $\mathbf{x}' = A\mathbf{x}$ using either (a) the method of elimination (write the equation as a system of linear first-order equations), or (b) the eigenvalue method.

Problem 3

Find a general solution to the differential equation $\mathbf{x}' = A\mathbf{x}$ where

$$A = \begin{bmatrix} 1 & -1 & 1 & 1 \\ 1 & 3 & -3 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}.$$

Problem 4

Let

$$F(t, x, \dot{x}) = x^2 + \frac{1}{2} t(t-1)\dot{x}^2,$$

and consider the problem

$$\min \int_2^3 F(t, x(t), \dot{x}(t))\, dt, \quad \text{subj. to} \quad x(2) = 0, \quad x(3) = \log\left(\frac{25}{27}\right). \qquad (1)$$

(a) Find the Euler equation (E) for (1).

(b) Show that (E) has a first degree polynomial solution x_1.

(c) Use reduction of order (see EP Exercise 2.2.36) to find another solution x_2 of (E) such that $x_2(t) = v(t)x_1(t)$, $t \in [2, 3]$.

(d) What is the general solution to (E)? Find the unique solution x_* to (E) that satisfies the endpoint conditions in (1).

(e) Decide whether x_* minimizes the integral in (1).

Good luck

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16 March 2015