MAT 2440 spring 2017— Exercises No 1

January 18, 2017

These are the exercises for the Wed Jan 25.

Exercises from the book

Chapter 1.8 pages 23–27: No 1, No 5 and No 7

Oppgaver

1.1. Write the third order equation

$$y''' + py'' + qy' + ry = 0$$

on matrix form. What is the determinant of the corresponding matrix? What is the characteristic polynomial?

1.2. Show that the general solution of of the affine system

$$\dot{x} = f(x, t) + g(t), \qquad (*)$$

can be written as the general solution of the associated linear system a particular solution of $(*)$ (that means just one).

1.3. If one also accepts negative solutions of the logistic equation, discuss the behavior when the initial value x_0 is negative.

1.4. There is a version of the logistic equation that incorporates a harvesting term h; that is, a constant term representing a continuos exploitation of the population. The equation the looks like

$$\dot{x} = rx(1 - x/k) - h.$$

Find the general solution and discussion the qualitative aspects.

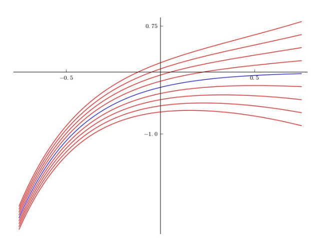
1.5. Determine the general solution of the equation

$$y' + y = \sin t.$$

Determine a solution such that $y(0) = 2$, and one with $y'(3\pi/2) = 1$.

1.6. Determine the general solution of the equation

$$y' - y = ae^{-bt} \qquad (1)$$



where a and b are positive constants. Show that $y_0(t) = -a/(b+1)e^{-bt}$ is a solution of the equation (1) and that it is the only solution that tends to a limit when $t \to \infty$. Assume that y is a solution such that for some t_0 it holds that $y(t_0) > y_0(t_0)$ then $y(t) \to \infty$ then $\lim_{t\to\infty} y(t) = \infty$ whereas $\lim_{t\to\infty} y(t) = -\infty$ for solutions satisfying $y(t_0) < y_0(t_0)$. Hint: Answer: $ce^x - a/(b+1)e^{-bx}$.

1.7. Find the general solution of

$$(1+x^2)y' + 3xy = 6x.$$

1.8. Let $f(t)$ be a function defined and twice differentiable in an interval I about zero, and at assume that f is positive. Let $y(t)$ be a solution of the equation

$$f(t)y' + 3ty = 6t.$$

Show that y has a local maximum at $t = 0$ if $y(0) > 2$ and a local minimum if $y(0) < 2$.

1.9. Find all solutions of

$$xy' = y.$$

Can you give a geometric explanation of the equation?

1.10. Determine all solutions of the differential equation

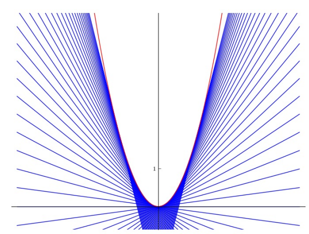
$$y' + \sqrt{y-c} = 0$$

where c is constant. Show that for any $y_0 > c$ there infinitely many continuously differentiable solutions satisfying $y(t_0) = y_0$. How many are twice differentiable?

1.11. Consider the equation

$$y = xy' - (y')^2, \tag{2}$$

which is one of the equations named *Clairot's equations*. If y is solution of (2) show that $y' = x/2$ or $y'' = 0$, and conclude that the solutions are either $y = x^2/4$ or $y = ax - a^2$ where a is an arbitrary constant. Show that the linear solutions are all the tangent to the parabola $y = x^2/4$. What condition on must (x_0, y_0) satisfy for (2) to have a solution with $y(x_0) = y_0$?



1.12. Show in detail that Wronskian of two linearly dependent functions vanishes.