UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Examination in:	MAT2700 — Introduction to mathematical finance and investment theory.
Day of examination:	Monday, December 12, 2005.
Examination hours:	09.00 – 12.00.

This examination set consists of 3 pages.

Appendices:	None.
Permitted aids:	None.

Make sure that your copy of the examination set is complete before you start solving the problems.

Problem 1.

Let a T-period market on a finite probability space Ω equipped with a strictly positive probability P be given by

- a bank account $B(t),\ t = 0, ..., T$,
- N stocks $S_1(t), ..., S_N(t),\ t = 0, ..., T$,
- a filtration $\mathbb{F} = \{\mathcal{F}_t\},\ t = 0, ..., T$ which models the information stream.

a) What is the definition of a trading strategy H in this market?

b) What is the (mathematical) definition of an arbitrage possibility in this market?

c) Let X be a contingent claim in this market. When do we say that X is attainable?

d) What is the definition of a complete market?

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Problem 2.

We define the following one-period market:

- a finite probability space $\Omega = \{\omega_1, ..., \omega_4\}$
- a probability P such that $P(\omega_i) > 0,\ i = 1, ..., 4$
- a bank account with $B_0 = 1$ and the interest rate $r = 1/8$
- two stocks $S_1(t), S_2(t),\ t = 0, 1$, given by the following table

	$S_1(0)$	$S_1(1)$	$S_2(0)$	$S_2(1)$
ω_1	4	9/4	5	45/8
ω_2	4	9/4	5	45/8
ω_3	4	27/4	5	9/2
ω_4	4	9/2	5	27/4

a) Determine all risk neutral prababilities.

b) Are there arbitrage possibilities in this market? Justify your answer.

c) Is the market complete? Justify your answer.

d) Determine all contingenent claims $X = (X_1, X_2, X_3, X_4)$ that are attainable. Here, X_i denotes $X(\omega_i)$, $i = 1, \ldots, 4$.

e) Is the "look back" option given by

$$X = \max\{0, S_2(0) - 4, S_2(1) - 4\}$$

attainable? If yes, what is its price?

Problem 3.

In this problem we will consider a 2-period binomial model given by

- a bank account with $B_0 = 1$ and interest rate $r = 0$
- a stock whose price dynamics is given by

$$S(t) = S(0)u^{N_t}d^{(t-N_t)}; \qquad t = 0, 1, 2.$$

We set the scaling factor for an up-movement of the stock $u = 2$ and the scaling factor for a down-movement of the stock $d = 1/2$. The initial price of the stock is $S(0) = 4$. The stochastic process N_t, $t = 0, 1, 2$, represents the number of "ups" of the stock price until time t.

Further, we assume the information stream to be modelled by the filtration generated by S, that is

$$\mathcal{F}_0 = \{\emptyset, \Omega\}$$
$$\mathcal{F}_1 = \big\{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}, \emptyset, \Omega\big\}$$
$$\mathcal{F}_2 = \text{all subsets of } \Omega.$$

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a) Is the market complete? Justify your answer.

b) Compute the generating strategy of the European call option

$$X_{\text{call}} = (S(2) - 3)^+$$

Verify that you get $V(0) = E_Q[X_{\text{call}}]$, where $V(0)$ is the initial value of the replicating portfolio and Q is a martingale probability.

c) What is the price process of the European put option

$$X_{\text{put}} = (3 - S(2))^+$$

d) What is the price process of the American put option given by

$$Y(t) = (3 - S(t))^+, \qquad t = 0, 1, 2\,.$$

Problem 4.

Let a one-period market be given by

- a finite probability space $\Omega = \{\omega_1, \dots, \omega_3\}$
- a probability P such that
$$P(\omega_1) = \tfrac{1}{2}; \quad P(\omega_2) = \tfrac{1}{3}; \quad P(\omega_3) = \tfrac{1}{6}$$
- a bank account with $B_0 = 1$ and interest rate $r = 0$
- two stocks $S_1(t), S_2(t)$, $t = 0, 1$, given by the following table

	$S_1(0)$	$S_1(1)$	$S_2(0)$	$S_2(1)$
ω_1	5	6	4	2
ω_2	5	8	4	6
ω_3	5	2	4	6

The unique risk neutral probability in this market is given by

$$Q(\omega_1) = 1/2$$
$$Q(\omega_2) = 1/6$$
$$Q(\omega_3) = 1/3$$

(you don't need to prove this).

In this market, we consider the following problem

$$\begin{cases} \max\limits_{H \in \mathbb{H}} E[u(V_1)] \\ V_0 = \nu \end{cases}$$

with utility function

$$u(x) = \tfrac{3}{2} x^{2/3} \qquad \text{and} \quad \nu = 15.$$

Here, $\mathbb{H}$ is the set of all self financing trading strategies, and $V(t)$, $t = 0, 1$, is the value process of the portfolio that corresponds to a trading strategy $H \in \mathbb{H}$. Find the value of the optimal portfolio with the help of the Lagrange method, and determine the corresponding optimal trading strategy.

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