UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: MAT3100 — Linear optimization.

Day of examination: Wednesday 16. June 2021.

Examination hours: 09:00 – 13:00.

This problem set consists of 2 pages.

Appendices: None.

Permitted aids: All.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

All 10 part questions will be weighted equally.

Problem 1 (Simplex method). We will consider the following LP.

$$\begin{array}{llcrcr} \max & -x_1 & - & 2x_2 & & \\ \text{s.t.} & -x_1 & - & x_2 & \leq & -2 \\ & x_1 & - & x_2 & \leq & 2 \\ & -x_1 & + & x_2 & \leq & 2 \\ & x_1 & + & x_2 & \leq & 6 \\ & & & x_1, x_2 & \geq & 0 \end{array} \tag{1}$$

a) Write down the dual problem. Write also both the primal and dual problems in matrix form.

b) Draw the feasible region of (1).

c) Write down the (initial) primal dictionary and the corresponding dual dictionary. Explain that the primal dictionary is not feasible, while the dual dictionary is.

d) Apply the simplex method to the dual problem, and write down the corresponding optimal dictionary for the primal problem. What are the optimal solutions to the primal and dual problems? Are they unique?

e) What is the optimal (x_1, x_2) if the objective in (1) is changed to $-x_1 - x_2$? Is the optimal solution unique?

Problem 2 (Convexity). What does it mean that a set $C \subseteq \mathbb{R}^n$ is convex, and that a function f from C to $\mathbb{R}$ is convex?
Show also that, if f is a convex function, then $h(x) = e^{f(x)}$ is also convex.
Hint: Use that $g(x) = e^x$ also is convex (you can use this fact without proving it).

Problem 3 (Game theory). Consider the matrix game with payoff matrix $A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$. In the following x denotes a (randomized) strategy

(Continued on page 2.)

for the column player, y a (randomized) strategy for the row player.

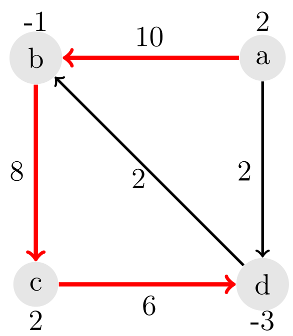
a) This question has two parts:

(i): Assume that the row player chooses strategy $y^* = (1, 0)$ (i.e., he always chooses the first item). What is the optimal strategy x for the column player (in order to maximize payoffs to himself)? What is the corresponding expected payoff?

(ii): Assume that the column player chooses strategy $x^* = (1, 0)$. What is the optimal strategy y for the row player (in order to minimize payoffs from himself)? What is the corresponding expected payoff?

b) Is it possible for the row player to choose a better strategy than y^*, i.e., so that his expected payoff is lower than what you obtained above?

Problem 4 (Network flow). Consider the minimum cost network flow problem based on the directed graph shown in the figure below.



The number associated with each directed edge is the cost per unit flow, and the number associated with each node is the supply at that node.

a) Let T_1 be the spanning tree consisting of the edges (a, b), (b, c), and (c, d) (this is indicated in red above). Compute the tree solution corresponding to T_1.

b) Use the network simplex method to find an optimal solution and optimal value for the flow problem.

Good luck!