$$\max \quad 3x_1 \quad + \quad 5x_2$$
$$\text{s.t.} \quad x_1 \qquad\qquad \leq 4$$
$$\qquad\qquad\qquad 2x_2 \quad \leq 12$$
$$\qquad 3x_1 \quad + 2x_2 \quad \leq 18$$

$$x_1, x_2 \geqslant 0$$

$$z = \quad 3x_1 \quad + \quad 5\underline{x_2} \qquad \boxed{1}$$
$$x_3 = 4 - x_1$$
$$x_4 = 12 \qquad\quad - 2x_2 \qquad \leftarrow \qquad x_2 = 6 - \frac{x_4}{2}$$
$$x_5 = 18 - 3x_1 \quad - 2x_2$$

$$z = 3x_1 + 30 - \frac{5}{2}x_4 = 30 + \frac{3x_1}{} - \frac{5}{2}x_4 \qquad \boxed{2}$$
$$x_3 = 4 \quad - x_1$$
$$x_2 = 6 \qquad\qquad - \frac{x_4}{2}$$
$$x_5 = 18 - 3x_1 - 12 + x_4 \qquad = 6 \quad - 3x_1 \quad + x_4 \qquad \leftarrow$$

$$x_1 = 2 + \frac{x_4}{3} - \frac{x_5}{3}$$

$$z \pm x_5 = 36 - \frac{3}{2}x_4 \qquad\qquad z = 36 - \frac{3}{2}x_4 - x_5 \qquad \boxed{3}$$
$$\begin{array}{l} -x_5 \\ 3x_3^{x_1} = 6 - x_4 \end{array} \qquad\qquad x_3 = 2 - \frac{x_4}{3} + \frac{x_5}{3}$$
$$x_2 = 6 - \frac{x_4}{2}$$
$$x_1 = 2 + \frac{x_4}{3} - \frac{x_5}{3}$$

$$z = 36 \quad , \quad x_1 = 2 \, , \quad x_2 = 6$$

$$\max \quad 5x_1 + 4x_2 + 3x_3$$

$$\text{s.t.} \quad 2x_1 + 3x_2 + x_3 \le 5$$

$$4x_1 + x_2 + 2x_3 \le 11$$

$$3x_1 + 4x_2 + 2x_3 \le 8$$

$$x_1, x_2, x_3 \ge 0$$

$$c = \begin{bmatrix} 10 \\ 12 \\ 29 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 4 & 1 \\ 2 & 1 & 2 \\ 5 & 2 & 2 \end{bmatrix} \quad b = \begin{bmatrix} 20 \\ 50 \\ 40 \end{bmatrix}$$

$$\boxed{\begin{array}{l} \max \ c^T x \\ s.t. \ \ Ax \leq b \\ \qquad x \geq 0 \end{array}} \qquad A^T = \begin{bmatrix} 0 & 2 & 5 \\ 4 & 1 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

$$\begin{array}{l} \max \ 10x_1 + 12x_2 + 29x_3 \\ s.t. \qquad \qquad 2x_2 + 5x_3 \leq 20 \\ \\ \qquad 4x_1 + 1x_2 + 2x_3 \leq 50 \\ \\ \qquad 1x_1 + 2x_2 + 2x_3 \leq 40 \\ \\ \qquad x_1, x_2, x_3 \geq 0 \end{array}$$

$$\boxed{\begin{array}{l} \eta = \ 10x_1 + 12x_2 + 29x_3 \\ \\ x_4 = 20 \qquad \quad - 2x_2 \ - 5x_3 \\ x_5 = 50 - 4x_1 \ \ - \ x_2 \ \ \ - 2x_3 \\ x_6 = 40 - x_1 \ \ - 2x_2 \ \ - 2x_3 \end{array}}$$

$$\forall j \qquad \underbrace{\sum_{i=1}^{3} A(i,j) \cdot x(i)}_{\parallel} \leq b(j)$$

$$\underbrace{\sum_{i=1}^{3} A^T(j,i) \, x(i)}_{\parallel}$$

$$\left(A^T x \right)_j$$