11.3

a row r dominates row s

$\hat{y} \in \mathbb{R}^m$ strategy for R

def. $\tilde{y} \in \mathbb{R}^m$:

$\tilde{y}_i = \hat{y}_i \quad \forall i \neq r, s$

$\tilde{y}_r = 0$

$\tilde{y}_s = \hat{y}_r + \hat{y}_s$

$$\Rightarrow \left[\tilde{y} = \hat{y} - \hat{y}_r e_r + \hat{y}_r e_s\right]$$

$\tilde{y}_i \geq 0 \ \forall i, \quad \sum_{i=1}^{m} \tilde{y}_i = 1$

$$\min_y \underbrace{\max_x y^T A x}_{= \max_x (A^T y)^T \cdot x} = \min_y \max_{j=1,\dots,m} y^T A e_j$$

$$\max_x b^T x, \quad x_i \geq 0, \quad \sum x_i = 1$$

$\Rightarrow x^* = e_{i_0}$, where

$i_0 = \text{argmax}(b)$

EX. $b = \begin{pmatrix} 1 \\ 5 \\ 10 \end{pmatrix}$

$\max b^T x = x_1 + 5x_2 + 10x_3$

s.t. $x_1 + x_2 + x_3 = 1$

$x_i \geq 0$

$\Rightarrow x^* = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = e_3$

$3 = \text{argmax}(b)$

$$f(y) := \max_x y^T A x = \max_{j=1,\dots,m} y^T A e_j$$

$$f(\tilde{y}) = \max_j \tilde{y}^T A e_j = \max_j (\hat{y} - \hat{y}_r e_r + \hat{y}_r e_s)^T \cdot A e_j$$

$$= \max_j (\hat{y}^T A e_j - \hat{y}_r a_{rj} + \hat{y}_r a_{sj})$$

r dominates s $\Rightarrow a_{rj} \geq a_{sj} \ \forall j$

$$\Rightarrow f(\tilde{y}) \leq \max_j (\hat{y}^T A e_j) = f(\hat{y})$$

b proved analogously

$$A = \begin{pmatrix} -6 & 2 & -4 & -7 & -5 \\ 0 & 4 & -2 & -9 & -1 \\ -7 & 3 & -3 & -8 & -2 \\ 2 & -3 & 6 & 0 & 3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -6 & 2 & -4 & -5 \\ 0 & 4 & -2 & -1 \\ -7 & 3 & -3 & -2 \\ 2 & -3 & 6 & 3 \end{pmatrix}$$

$$\begin{pmatrix} -6 & 2 & -4 & -5 \\ -7 & 3 & -3 & -2 \\ 2 & -3 & 6 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -4 & -5 \\ 3 & -3 & -2 \\ -3 & 6 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -4 & -5 \\ -3 & 6 & 3 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 2 & -4 \\ -3 & 6 \end{pmatrix}$$