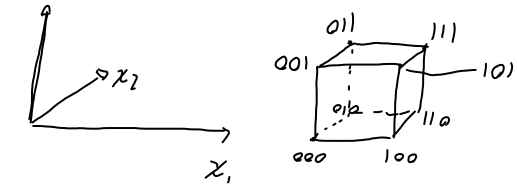
9 P is an hypercube whose extreme points are

$V = \{ (\varepsilon_1, .., \varepsilon_n) \text{ s.t. } \varepsilon_i \in \{0, 1\} \forall i \}$



$\Rightarrow$ we look for $\max \{ c^T \varepsilon \text{ s.t. } \varepsilon = (\varepsilon_1, .., \varepsilon_n)$
$\varepsilon_i \in \{0,1\} \}$

$I := \{ \text{indices s.t. } c_i > 0 \} \subseteq \{1, .., n\}$

$J := \{1, .., n\} \setminus I$

Then $\max \{ c^T x : x \in P \} = \max \{ c^T x : x \in V \}$

$= \sum_{i \in I} c_i$ and it is attained in

$\bar{x} = (\bar{\varepsilon}_1, \dots, \bar{\varepsilon}_n)$ where $\bar{\varepsilon}_i = \begin{cases} 1 & \text{if } i \in I \\ 0 & \text{if } i \in J \end{cases}$

$a_i \le x_i \le b_i$, $a_i \le b_i$

$V := \{ (\varepsilon_1, .., \varepsilon_n) \text{ s.t. } \varepsilon_i \in \{a_i, b_i\} \}$

$\max \{ c^T x : x \in P \} = \max \{ c^T x : x \in V \}$

$= \max \{ c^T x : x = (\varepsilon_1, .., \varepsilon_n), \varepsilon_i \in \{a_i, b_i\} \}$

- if $c_i > 0$, $c_i b_i \geq c_i a_i$
- otherwise, $c_i b_i \le c_i a_i$

$\Rightarrow \max \{ c^T x : x \in P \} = \sum_{i \in I} c_i b_i + \sum_{j \in J} c_j a_j$

and this is attained in

$\bar{x} = (\bar{\varepsilon}_1, \dots, \bar{\varepsilon}_n)$, $\bar{\varepsilon}_i = \begin{cases} b_i & \text{if } i \in I \\ a_i & \text{if } i \in J \end{cases}$