$z_{uv} = c_{uv} + y_u - y_v$

$z_{cf} = 3 + 0 - 23 = -20$

$z_{ef} = 8$

$z_{ac} = 24$

$z_{ae} = -3$

$z_{ab} = 10$

$z_{ha} = 3$

---

a) ad = entering arc
ag = leaving arc

$\tilde{x}_{ag} = 1 - 1 = 0$

$\tilde{x}_{ge} = 14 - 1 = 13$

$\tilde{x}_{eb} = 14 - 1 = 13$

$\tilde{x}_{bd} = 6 - 1 = 5$

$\tilde{x}_{ad} = 0 + 1 = 1$

$z_{ad} = -13$

$\tilde{z}_{ag} = z_{ag} + 13 = 0 + 13 = 13$

$\tilde{z}_{dc} = z_{dc} - 13 = 18 - 13 = 5$

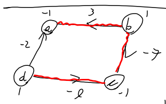
$\tilde{z}_{ad} = 0$

$\tilde{z}_{ba} = z_{ba} \mp 13 = -3$

$\tilde{z}_{ea} = 0$

$\tilde{z}_{af} = 27$

$\tilde{z}_{fc} = -23$



$x_{ba} = 1$

$x_{bc} = 0$

$x_{dc} = 1$

$y_a = 0$

$y_b = -3$

$y_c = -10$

$y_d = -1$

$\tilde{x}_{ba} = 0$

$\tilde{x}_{bc} = 1$

$\tilde{x}_{dc} = 0$

$\tilde{x}_{da} = 1$

$z_{da} = c_{da} + y_d - y_a = -2 - 1 + 0 = -3$

da enters, ba leaves

$\tilde{y}_a = 0 \quad \tilde{y}_d = 2$

$\tilde{y}_c = -7 \quad \tilde{y}_b = 0$

$\Rightarrow \tilde{z}_{ba} = c_{ba} + \tilde{y}_b - \tilde{y}_a = 3 + 0 - 0 = 3 \geq 0$

14.12

$\tilde{A}$ is the incident matrix without the last row.

Suppose we have n vertices and m edges.

$\tilde{A} \in \mathbb{R}^{n-1, m}$.

B inv. sq. submatrix of $\tilde{A}$

$B \in \mathbb{R}^{n-1, n-1}$

$\Rightarrow$ the arcs corresponding to columns in B are $n-1$ $\Rightarrow$

they form a spanning tree iff they don't have a cycle.

Suppose that some columns of B $(c_1, c_2, \dots, c_k)$ correspond to a cycle

Consider $v = \sum_{i=1}^{k} c_i \in \mathbb{R}^{n-1}$

If j is not a vertex in the cycle,

$(c_i)_j = 0 \quad \forall i = 1, \dots, k$

$\Rightarrow v_j = 0$

If j is a vertex in the cycle,

j is the initial vertex of one edge $c_{\hat{i}}$

and the final vertex of $c_{\bar{i}}$,

and it is not incident to any other edge in the cycle.

$v_j = \left(\sum_{i=1}^{k} c_i\right)_j = \sum_{i=1}^{k} (c_i)_j = (c_{\hat{i}})_j + (c_{\bar{i}})_j$

$= 1 - 1 = 0$

$\Rightarrow v = 0 \Rightarrow \sum_{i=1}^{k} 1_i = 0$

$\Rightarrow c_1, \dots, c_k$ are l.d. $\Rightarrow B$ is not inv.

14.14 INPUT : V (vertex set), E (edge set)

OUTPUT : S (edge set of the spanning tree) or message

```
n = length(V)
S = [ ]
W = [ ]
W.add (V(1))
for (i = 1:n-1)
    for e in E
```