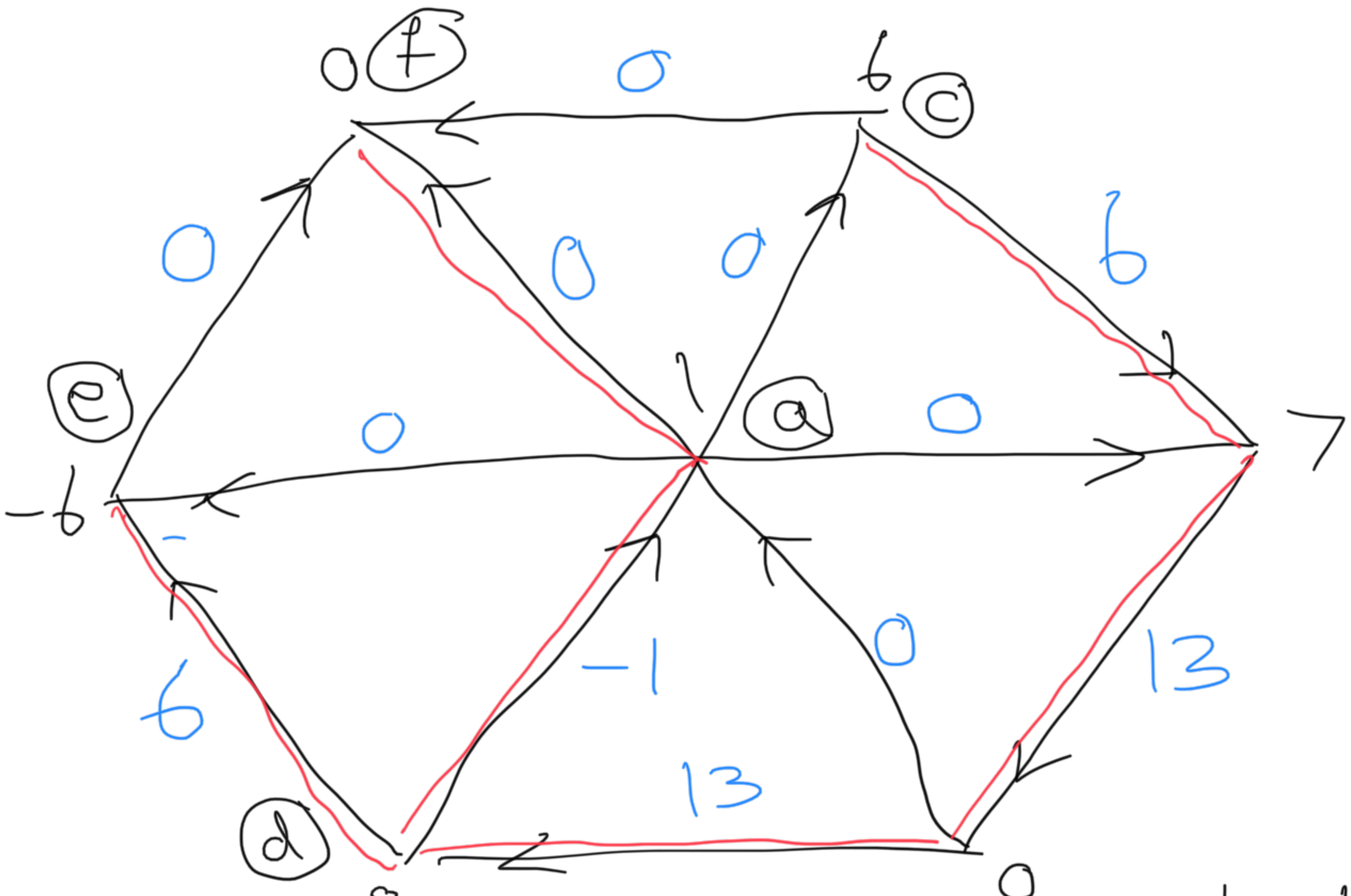
Exercises, Network Flows.

Ex 14·3 Directed graph.



Find the x_{uv} given the b_v's

Compute the basic x's

use outflow $-$ inflow $= b_v$

at vertex v.

Choose e.

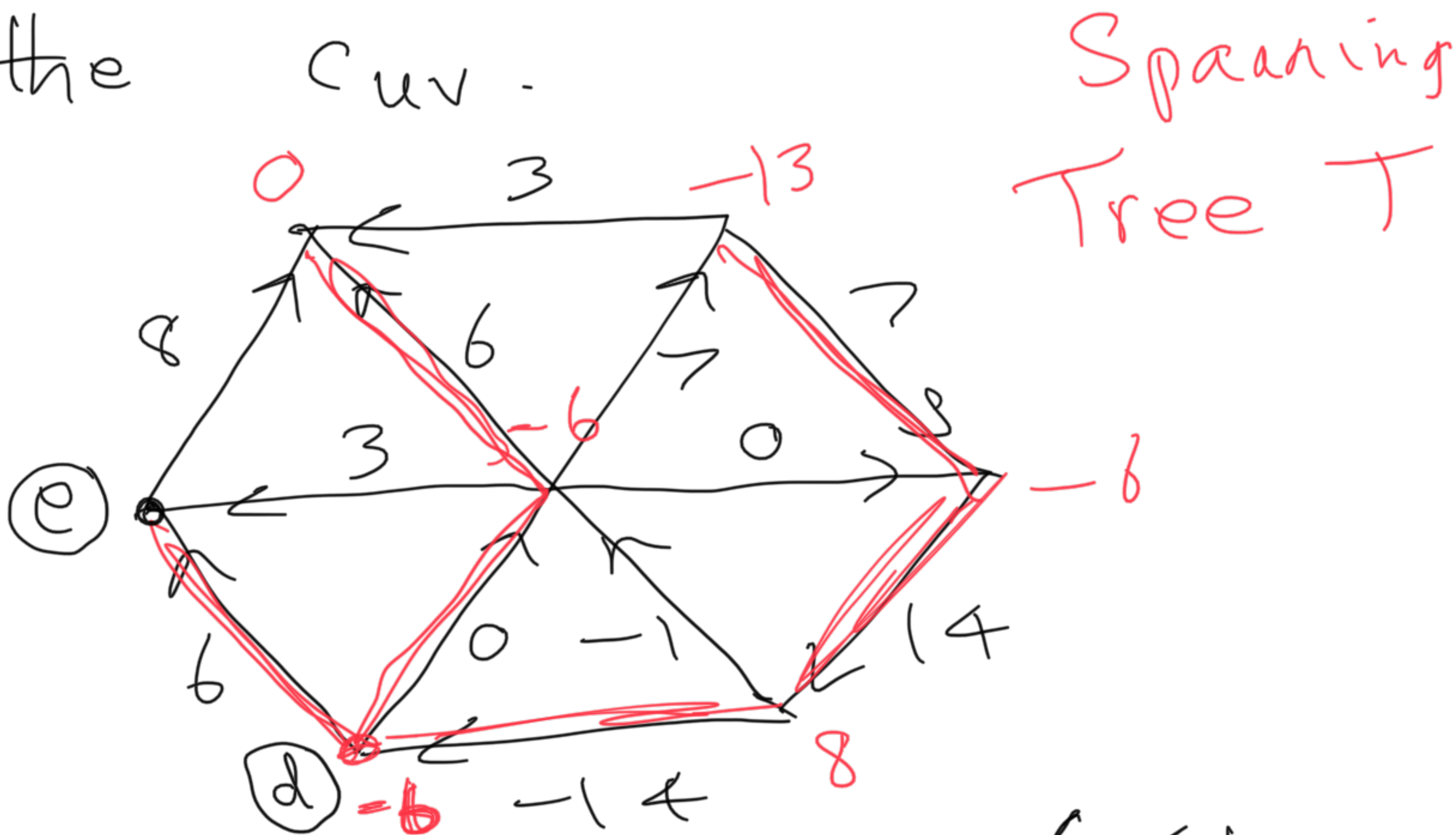
$$x_{ef} - (x_{ae} + x_{de}) = b_e$$

$$0 - (0 + x_{de}) = -6$$

$$\Rightarrow x_{de} = 6.$$

$x_B \not\geq 0$ So this x is not feasible.

(b) Compute y_v's from the c_{uv}.



Let $y_e = 0$. $\left(\sum_{v \in V} b_v = 0\right)$

Remaining y_v are uniquely determined. Leaf elimination

Use ... $y_v = c_{uv}$

use $y_v - y_u = c_{uv}$.
for each edge (u,v) in T.

So $\quad y_e - y_d = c_{de}$

$$0 - y_d = 6$$

$$\Rightarrow \quad y_d = -6.$$

(c) Compute Z_{uv} for each (u,v) not in T.

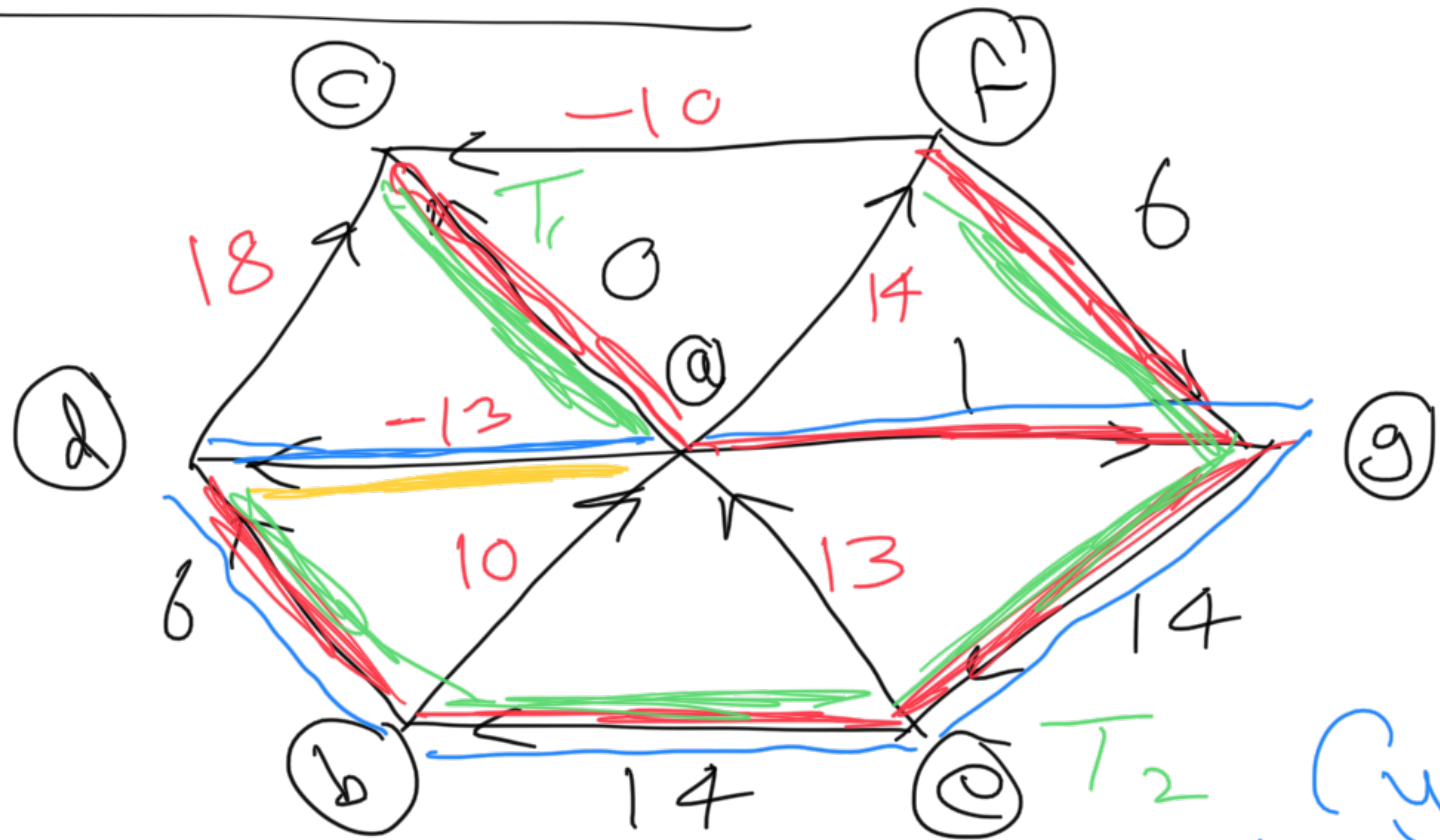
$$Z_{uv} = c_{uv} - (y_v - y_u)$$

$c_{ef} = 3 \qquad y_f = 0, \quad y_e = -13$

$$\Rightarrow \quad Z_{ef} = 3 - (0 - (-13))$$

$= 5 - 15 = -10.$

Exercise 14.4



leave
yellow
enter

T_1, T_2

Cycle
add (a,d) to T.

x_B $\qquad$ z_N.

(a) Which variable x_{ij} enters the basis?

Answer: smallest z_{ij}:

$$\bar{z}_{ad} = -13.$$

So x_{ad} enters the basis.

(b) Which x variable leaves?

Let $x_{ad} = \varepsilon$. Then (a, d) forms a cycle with T:

$$a, d, b, e, g, a.$$

$$\tilde{x}_{ad} = \varepsilon$$

$$\tilde{x}_{bd} = x_{bd} - \varepsilon = 6 - \varepsilon$$

$$\tilde{x}_{eb} = x_{eb} - \varepsilon = 14 - \varepsilon$$

$$\tilde{x}_{ge} = x_{ge} - \varepsilon = 14 - \varepsilon$$

$$\tilde{x}_{ag} = x_{ag} - \varepsilon = 1 - \varepsilon$$

Increase until $\varepsilon = 1$ Then

$\tilde{x}_{ag} = 0$. Then x_{ag} leaves.

$$\tilde{x}_{ad} = 1$$
$$\tilde{x}_{bd} = 5$$
$$\tilde{x}_{eb} = 13$$
$$\tilde{x}_{ge} = 13$$
$$\tilde{x}_{ag} = 0$$

(c) Compute new slack (dual) variables z_{uv}.

Only change z values on edges that bridge the

two subtrees formed
by removing (a,g) from T.
Those edges that bridge
in same direction as entering
(a,d) are decreased by
$Z_{ad} = -13$. Else increased
by Z_{ad}.

$T_1 \to T_2$ $\tilde{Z}_{ag} = Z_{ag} - (-13) = 0+13 = 13$
$T_2 \to T_1$ $\tilde{Z}_{dc} = Z_{dc} + (-13) = 18-13 = 5$
$\tilde{Z}_{ad} = 0$

- - - etc.