MAT3100 - Compulsory exercise 1 of 2, 2022

February 27, 2022

Deadline

Thursday 17. march, 2022, 14:30, in Canvas (canvas.uio.no).

Instructions

Note that you have one attempt to pass the assignment. This means that there are no second attempts.

For courses on bachelor level, you can choose between scanning handwritten notes or using a typesetting software for mathematics (e.g. LaTeX). Scanned pages must be clearly legible. For courses on master level the assignment must be written with a typesetting software for mathematics. It is expected that you give a clear presentation with all necessary explanations. The assignment must be submitted as a single PDF file. Remember to include any relevant programming code and resulting plots and figures, in the PDF-file.

All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, you may be asked to give an oral account.

Application for postponed delivery: If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) before the deadline. Note that teaching staff can not grant extensions.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination. To pass this compulsory exercise you need to make real attempts on all parts, and at least 50% needs to be answered satisfactory.

For complete guidelines for delivery of compulsory exercises, see:

www.uio.no/studier/admin/obligatoriske-aktiviteter/mn-math-oblig.html

Good luck!

Consider the problem

$$
\begin{array}{rrcrcl}
\max & x_1 & + & 2x_2 & & \\
\text{s.t.} & -x_1 & + & x_2 & \leq & 1 \\
 & x_1 & - & x_2 & \leq & 1 \\
 & x_1 & & & \leq & 2 \\
 & & & x_2 & \leq & 2 \\
 & & & x_1, x_2 & \geq & 0
\end{array}
\tag{1}
$$

a)

Write down a 4×2-matrix A and vectors $\mathbf{b}$ and $\mathbf{c}$ so that (1) can be written on the form

$$
\begin{aligned}
\max\ & \mathbf{c}^T \mathbf{x} \\
\text{s.t. } & A\mathbf{x} \leq \mathbf{b} \\
& \mathbf{x} \geq \mathbf{0}
\end{aligned}
$$

b)

Solve the problem using the Simplex method and the largest coefficient rule.

c)

Sketch the feasible region of the problem (1). In your drawing, indicate the (directed) path taken by the basic feasible solutions in the iterations of simplex from **b)**. If you instead applied Blands rule, what would this path be?

d)

We consider the same constraints as in **a)**, but change the objective function to x_2. Find an optimal solution to this modified problem. Is the optimal solution unique? If not, find an expression for the general optimal solution. You can use your drawing of the feasible region from **c)** as a guide.

For the last three questions, we return to the original objective function, i.e., the problem (1).

e)

Write down the dual problem of (1), as well as the initial dual dictionary. Is this dictionary feasible?

f)

Write down the dual dictionaries corresponding to the primal dictionaries you obtained when applying the simplex method in **b)**. What are the basic solutions obtained by these dual dictionaries?

g)

Find the slack variables for the primal and dual optimal solutions. State what is meant by complementary slackness, and verify that this property holds for the optimal solutions you have found.