MAT3100 - Compulsory exercise 2 of 2, 2022

April 25, 2022

Deadline

Thursday 12. may, 2022, 14:30, in Canvas (canvas.uio.no).

Instructions

Note that you have one attempt to pass the assignment. This means that there are no second attempts.

For courses on bachelor level, you can choose between scanning handwritten notes or using a typesetting software for mathematics (e.g. LaTeX). Scanned pages must be clearly legible. For courses on master level the assignment must be written with a typesetting software for mathematics. It is expected that you give a clear presentation with all necessary explanations. The assignment must be submitted as a single PDF file. Remember to include any relevant programming code and resulting plots and figures, in the PDF-file.

All aids, including collaboration, are allowed, but the submission must be written by you and reflect your understanding of the subject. If we doubt that you have understood the content you have handed in, you may be asked to give an oral account.

Application for postponed delivery: If you need to apply for a postponement of the submission deadline due to illness or other reasons, you have to contact the Student Administration at the Department of Mathematics (e-mail: studieinfo@math.uio.no) before the deadline. Note that teaching staff can not grant extensions.

All mandatory assignments in this course must be approved in the same semester, before you are allowed to take the final examination. To pass this compulsory exercise you need to make real attempts on all parts, and at least 50% needs to be answered satisfactory.

For complete guidelines for delivery of compulsory exercises, see:

www.uio.no/studier/admin/obligatoriske-aktiviteter/mn-math-oblig.html

Good luck!

Consider the LP problem

$$
\begin{array}{rrrrrrcl}
\max & 3x_1 & + & 2x_2 & + & x_3 & & \\
\text{s.t.} & 3x_1 & + & 4x_2 & + & x_3 & \leq & 6 \\
 & 2x_1 & + & x_2 & + & 3x_3 & \leq & 5 \\
 & & & & & x_1, x_2, x_3 & \geq & 0
\end{array}
\tag{1}
$$

a)

Solve the problem using the Simplex method. What is the optimal value? Is the optimal solution unique?

b)

There are six extreme points in the feasible region of (1). Find them.
Hint: Use Proposition 7 in mini-introduction to convexity, and do as in Example 5 there.

c)

Find two points $\mathbf{x}_1$ and $\mathbf{x}_2$ so that the set of optimal solutions to (1) can be written as $\text{conv}(\mathbf{x}_1, \mathbf{x}_2)$ (recall that conv denotes the convex hull of the points). Explain also that the feasible region of (1) equals the convex hull of the six points you found in **b)**.

d)

Show that $P \cap Q$ is a polyhedron when P and Q are polyhedra. Show also that $P \cap Q$ is a polytope when P and Q are polytopes.

In the last two exercises, $A + B$ is the set consisting of all points on the form $a + b$ with a in the set A, b in the set B.

e)

Show that $A + B$ is a convex set when A and B are convex sets.

f)

Let $A, B \subseteq \mathbb{R}^n$. Prove that $\text{conv}(A + B) = \text{conv}(A) + \text{conv}(B)$.
Hint: it is useful to consider the sum $\sum_{j,k} \lambda_j \mu_k (a_j + b_k)$ where $a_j \in A$, $b_k \in B$ and $\lambda_j \geq 0$, $\mu_k \geq 0$ and $\sum_j \lambda_j = 1$ and $\sum_k \mu_k = 1$.