Oblig 1

Øyvind Ryan

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Consider the problem

$$
\begin{array}{rrcrcrcl}
\max & x_1 & + & x_2 & + & 2x_3 & & \\
\text{s.t.} & x_1 & + & x_2 & + & x_3 & \leq & 5 \\
 & x_1 & & & & & \leq & 2 \\
 & & & x_2 & & & \leq & 2 \\
 & & & & & x_3 & \leq & 2 \\
 & x_1, & & x_2, & & x_3 & \geq & 0
\end{array}
\tag{1}
$$

a)

Write down a 4×3-matrix A and vectors $\mathbf{b}$ and $\mathbf{c}$ so that (1) can be written on the form

$$
\begin{aligned}
\max\ & \mathbf{c}^T \mathbf{x} \\
\text{s.t.}\ & A\mathbf{x} \leq \mathbf{b} \\
& \mathbf{x} \geq \mathbf{0}
\end{aligned}
$$

b)

Solve the problem using the Simplex method and Blands rule. Is the optimal solution unique? If not, write down the general optimal solution.

c)

Write down all basic feasible solutions that the simplex method encountered in **b)**. If you instead applied the largest coefficient rule, what would the corresponding list of basic feasible solutions then be?

d)

Write down the dual problem of (1), as well as the dual dictionaries corresponding to the initial and optimal dictionaries from **b)**. What is the optimal solution, and is it unique?

e)

State what is meant by complementary slackness, and verify that this property holds for the optimal solutions you have found for the primal and dual problems in **b)** and **d)**.

f)

We consider the same constraints as in **a)**, but change the objective function to $x_1 + 2x_2 + 2x_3$. Find an optimal solution to this modified problem. Is the optimal solution unique now?