Oblig 2

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Problem 1 (Game theory)

For this first problem you may need to consult the lecture notes on game theory from 31. march (see the schedule of the course), as the material on pure minmax- and maxmin strategies can not be found in the book. We consider the matrix game with payoff matrix

$$A = \begin{pmatrix} -2 & -2 & 2 \\ -2 & 0 & 1 \\ 1 & -2 & 0 \end{pmatrix}$$

a)

Find the best pure strategies for the row- and column player (i.e., the minmax and maxmin strategies), and their corresponding payoffs (V^* and V_*). Show that the game does not have a value (i.e., no pure maxmin- and minmax strategies).

b)

State the Minimax Theorem for matrix games. Explain why the pure strategies found above are not optimal, so that both players should consider using a randomized strategy instead.

c)

Let $\mathbf{x}^* = (1/4, 0, 3/4)$ and $\mathbf{y}^* = (0, 1/4, 3/4)$ be randomized strategies for the column player and row player, respectively. Prove that these are optimal strategies, and find the value of the game. Is the game fair? If not, which player wins in the long run?

Problem 2 (Convexity)

For this second problem, you need to consult the notes ”mini-introduction to convexity”, found on the course webpage.

a)

Let f be a convex function, and α a scalar. Show that the ”sub-level” set $C = \{x : f(x) \leq \alpha\}$ is convex.

b)

Let f and g be convex functions. Show that $h(x) = \max(f(x), g(x))$ also is a convex function.

c)

Show that

$$\text{conv}\{\mathbf{0}, \mathbf{e}_1, -\mathbf{e}_1, \mathbf{e}_2, -\mathbf{e}_2, ..., \mathbf{e}_n, -\mathbf{e}_n\} = \{\mathbf{x} \in \mathbf{R}^n : \sum_{j=1}^{n} |x_j| \leq 1\}$$

(the left hand side is the convex hull of $2n + 1$ vectors in $\mathbf{R}^n$. The $\mathbf{e}_i$ are the standard unit basis vectors in $\mathbf{R}^n$)