UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Examination in	MAT3100 — Linear optimization
Day of examination:	07 June 2024
Examination hours:	1500 – 1900
This exam paper consists of 3 pages.	
Appendices:	None
Permitted aids:	None

Please make sure that your copy of the exam paper is complete before you attempt to answer anything.

All 10 part questions will be weighted equally.

Problem 1 Simplex method

Consider the LP problem (P)

$$\begin{aligned} \text{maximize} \quad & x_1 + 2x_2 \\ \text{subject to} \quad & 3x_1 + x_2 \le 9, \\ & -x_1 + x_2 \le 1, \\ & x_1, x_2 \ge 0. \end{aligned}$$

1a

Use the simplex method to find an optimal solution and the corresponding objective value.

1b

Make a plot of the feasible region for (P) and indicate your optimal solution.

1c

(i) Let (P′) be the LP problem formed by adding the constraint, $x_1 - 2x_2 \le 4$, to (P). What is the optimal objective value of (P′)?
(ii) Let (P″) be the LP problem formed by replacing the objective function of (P) by $x_1 + 3x_2$. What is the optimal objective value of (P″)?

1d

What is the dual problem (D) of (P)?

(Continued on page 2.)

1e

What is an optimal solution to (D) and what is the corresponding optimal value?

Problem 2 Standard form

Convert the LP problem

$$\begin{array}{lllll} \text{minimize} & 3x_1 & -4x_2 & -2x_3 & \\ \text{subject to} & -x_1 & +2x_2 & +3x_3 & \geq 2, \\ & 2x_1 & -x_2 & & = 5, \\ & & x_1, x_3 \geq 0 & & \end{array}$$

into standard form (the form suitable for the simplex algorithm). Note that $x_2 \in \mathbb{R}$ is a free variable. What form of the simplex algorithm will be required to solve it? (do not try to solve it).

Problem 3 Convexity

3a

Recall that a polytope P is the convex hull of a finite set of points $\mathbf{x}_1, \ldots, \mathbf{x}_t \in \mathbb{R}^n$. Show that P is convex.

3b

Let A_1 and A_2 be $m \times n$ matrices and let $\mathbf{b}_1$ and $\mathbf{b}_2$ be vectors in $\mathbb{R}^m$. Show that the set

$$S := \{\mathbf{x} \in \mathbb{R}^n : A_1\mathbf{x} \leq \mathbf{b}_1, A_2\mathbf{x} = \mathbf{b}_2\}$$

is convex (if it is non-empty).

Problem 4 Network flow

Consider the minimum cost network flow problem based on the directed graph shown in the figure. The number associated with each directed edge (i, j) is its cost per unit flow, $c_{i,j}$, and the the number associated with each node i is its supply, b_i.

4a

Let T_1 be the spanning tree formed by the edges (a, c), (c, b), (d, b), shown in bold in the figure. Compute the tree solution x corresponding to T_1.

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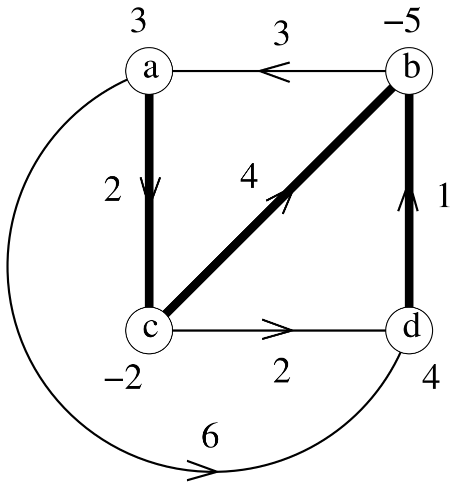

4b

Use the network simplex method to find an optimal solution and optimal value for the flow problem.

Good luck!

(Continued on page 4.)