Gaussian elimination of lower triangular matrix

Given a lower triangular matrix $A \in \mathbb{R}^{n \times n}$ we know that it requiers $cn^2 + O(n)$ operations to perform Gaussian elimination on the matrix (A, b) $b \in \mathbb{R}^n$
We want to find the constant c.

Note that I only count the number of multiplications.
See sec 1.8 in book for discussion on this.

We have that

$$(A, b) = \begin{bmatrix} a_{11} & & & & b_1 \\ a_{21} & a_{22} & & & b_2 \\ a_{31} & a_{32} & a_{33} & & \vdots \\ \vdots & & & \ddots & \end{bmatrix}$$

$$\begin{bmatrix} \vdots & & \ddots & & \vdots \\ a_{n1} & \cdots & \cdots & a_{nn} & b_n \end{bmatrix}$$

In the first step we use row 1 to eliminate all elements in column 1 by computing

$$s_i = \frac{a_{i1}}{a_{11}} \Big\} 1$$

$$(a_{i1} \ldots a_{ii}\ 0 \cdots 0\ b_i) - \underbrace{s_i\ (a_{11}}_{1}\ 0 \cdots\ 0\ \underbrace{b_1}_{1})$$

for $i = 2, \ldots n$.

Each iteration requiers 3 multiplications, i.e., a total of $3(n-1)$ operations.

To eliminate all elements under the main diagonal in column 2 we use the same procedure

This will cost $3(n-2)$ operations. More generaly we see that it will cost

$$3(n-1)+3(n-2)+\cdots+3\cdot 2+3\cdot 1$$

operations to remove all elements under the main diagonal.

Thus the total cost is

$$\sum_{i=1}^{n-1} 3i = \frac{3(n-1)n}{2} = \frac{3}{2}n^2+O(n)$$