Householder reflection

$$B = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} = (b_1, b_2)$$

where $b_i \in \mathbb{R}^2$ are the columns of B.

Let $u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \in \mathbb{R}^2$

From sec 2.8 in the book we have that

$$u_2 = (b_1)_2 = 2$$

and $u_1 = (b_1)_1 + \|b_1\|$

$$= 1 + \sqrt{1^2 + 2^2} = 1 + \sqrt{5}$$

This means that $u = \begin{pmatrix} 1+\sqrt{5} \\ 2 \end{pmatrix}$

and $H = I - \dfrac{2uu^T}{u^T u}$

Let $HB = R$. By construction

of u, we know that R is an upper triangular matrix.
Let $Q = H^T$. Then

$$B = H^T R = QR$$ is a QR factorization of B.

Likewise we now consider

$$A = \begin{pmatrix} 0 & 2 \\ 1 & 3 \end{pmatrix} = (a_1 \;\; a_2),$$

$a_i \in \mathbb{R}^2$ are columns of A.

Let $u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \in \mathbb{R}^2$.

We have that

$$u_2 = (a_1)_2 = 1$$

$$u_1 = (a_1)_1 + \|a_1\| = 1$$

i.e., $u = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and

$$H = I - \frac{2uu^T}{u^Tu} = I - \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$HA = R$ where R is upper triangular

$\Rightarrow$ $H^T R$ is a QR factorization of A.

In this solution draft I have not gone through all the theory for why these computations work. It is recomended to read sec. 2.8 to get a better understanding.